Author’s Response

Dear Editor and Reviewers:

Thanks very much for your careful review and constructive suggestions on the manuscript “Estimation of flow velocity for a debris flow via the two-phase fluid model” (npg-2014-39). We have revised the manuscript carefully according to reviewers’ comments. The detailed revisions are listed below based on reviewer’s response point by point.

1. Response to the first reviewer

Qu. 1 Introduction: In the second § it does not become clear whether a two layer system (slurry and dense fluid-solid mixture) is looked at.

The three stated assumptions are extremely restricting:

(1) No geometric deformation of the moving mass is possible

(2) What is meant by that ‘no external materials are involved’ [do you mean erosion and deposition] and there are ‘no transformations between the solid and the liquid phases’ [Do you mean that no phase changes occur?]

(3) Steadiness. This is almost never the case.

Ans. In this article, “the solid phase” denotes “the solid phase particles”, “the liquid phase” denotes “the liquid phase slurry”. We have change “the solid phase” and “the liquid phase” into “the solid phase particles” and “the liquid phase slurry” respectively. ‘no external materials are involved’, that is, both sides bank slope of the debris flow groove doesn’t supply materials, the debris flow keep balance in the groove. ‘no transformations between the solid and the liquid phases’, which means that no transformation between the solid phase particles and liquid phase slurry. To deal with the velocity of a debris flow is a quite complicated process. For convenience of calculations, we assume that a debris flow is steady (see [1-3]).

Qu. 2 For the presentation of eqs. (1)-(4) it should be said that the two phases are density preserving and the mixture is saturated.

Ans. This is explained in our assumption. A homogeneous flow and no transformation between the solid phase particles and liquid phase slurry guarantee that the two phases are density preserving, and no external materials are involved guarantees that the mixture is saturated.

Qu. 3 Equations (5), (6): I do not understand the term ‘surface forces’ I would interpret fs and ff as interaction forces between the phases, and fs+ ff= 0 would be required. Is this satisfied?

Ans. In the movement of a debris flow, taking which in a unit volume as the research object, which is said to be control volume. On surface of control volume, there exists the acting forces from debris flow outside control volume, it is said to be surface forces. Here, the surface forces (fsx and ftx) in a unit volume beyond pressure are considered. Since the debris flow is divided two phases: the solid phase particles and liquid phase slurry, the surface forces of the solid phase fsx on control volume is divided into two parts: the traction of liquid phase slurry outside control volume, ftx1, and the force from solid particles outside control volume, fsx2; the surface forces of the liquid phase ftx on control volume is divided into two parts: the resistance from particles outside control volume, ftx1, and the resistance from liquid phase slurry outside control volume, ftx2. It can be
seen that $f_{sx1}$ and $f_{fx1}$ are a pair of interaction forces between the phases, and they satisfy $f_{sx1} + f_{fx1} = 0$.

**Qu. 4** Equation (7): Is $v$ the barycentric velocity? Is there any literature reference for the value of $k$, given after eq. (8)?
It is not clear in this context what a ‘viscous debris flow’ is against a ‘thin debris flow. Please be precise.

**Ans.** Yes, $\nu$ is the barycentric velocity, the non-uniform coefficient $k$ can be found in [4, pp. 177-178] and we have added the references in our manuscript. In [4, pp. 177-178], the non-uniform coefficient $k$ is about 2.4–3.0 for a viscous debris flow; $k$ is about 3.0–3.5 for a transitional debris flow; $k$ is about 3.5–4.0 for a thin debris flow. We have changed “the non-uniform coefficient $k$ is about 2.4–3.0 for a viscous debris flow, whereas $k$ is about 3.5–4.0 for a thin debris flow” into “the non-uniform coefficient $k$ is about 2.4–3.0 for a viscous debris flow; $k$ is about 3.5–4.0 for a thin debris flow (see [4])”. We mainly consider both the viscous and thin debris flow, the transitional debris flow don’t been involved.

**Qu. 5** Equation (11): Is $\bar{v}$ in this equation the same as $v$?

**Ans.** $\nu$ is the velocity of debris flow, whereas $\bar{v}$ is the velocity of the debris flow in $x$ direction.

**Qu. 6** Text and equations between eqs. (13)-(18): This text needs to be revised. It does not become clear what ‘outside control volumes’ etc. mean. Perhaps the authors mean the volume of the pore space or the ‘grain area wetted by the fluid’. In the text from (13)-(18) twelve articles ‘the’ are missing and after eq. (15) ‘the pressure difference’ is NOT ‘generated’ but ‘is acting’. Moreover, it is not clear, how the two choices of $P_0$ and $T_0$ in the un-numbered equations are connected. No hints or references are given.

**Ans.** The explanation regarding control volume can be seen in the response of Qu. 3. We have added the articles ‘the’ and replace ‘generated’ by ‘is acting’. The two choices of $P_0$ and $T_0$ in the un-numbered equations can been seen in [5,6].

**Qu. 7** What is a ‘framboid’? (top on page 5) Please also explain the meaning of $l$. Is it the boundary layer thickness around the grains?

**Ans.** We have changed ‘framboid’ into ‘eddy’, $l$ is the mixing length among flow layers in the debris flow body, that is, the moving distance of eddies in the liquid phase slurry, which is caused by the effect of the fluctuating velocity.

**Qu. 8** It is stated on top of page 6 down to eq. (23) that ‘the turbulence parameter $\eta$ and the velocity profile parameters $a$, $b$, $c$ must be determined experimentally determined. But how is this done? Please explain, the formulas ought to be useful.

**Ans.** The turbulence parameter $\eta$ can be taken as Karman constant [6]. The velocity profile parameters $a$, $b$, $c$ are obtained through the experimental simulation based on the analysis and study of the sampled debris flow deposits.

**Qu. 9** Text between eqs. (23) and (24). Here all of a sudden ‘the velocity of the solid phase in the $y$-direction and the effect of the turbulence in the slurry are ignored’. Everything from eqs. (24)-(35) is then restricted to this simplification. Can we then simply forget the text between eq. (20) and eq. (23)?
In the process of a debris flow movement, including the forward flow along the debris flow groove and the vertical turbulent in the debris flow body, however, from velocity analysis of the debris flow, especially the impact and abrasion of the debris flow for the controlling structure and bank slope, the forward movement of the debris flow is mainly concerned (see [6]). Hence, we mainly consider the velocity of the solid phase in the x-direction, i.e. we can regard η as 0, then $f_{tx2} = aμd_0^2 + (τ_B + μb)d_0$. Further, for the convenience of calculation, we will take linear distribution of velocity of the liquid phase slurry with respect to y [6] as an example. So that, it follows that $f_{tx2} = (τ_B + μb)d_0$.

Qu. 10 Can you explain in a few words how eq. (32) is solved to obtain eq. (33)?

Ans. Let $y = \frac{ρfv^2 - ρsv^2}{2}$. $A = -\frac{4k}{(2k+1)(1-\varphi)c_s}$, $B = -\frac{1}{2k+1}[(2ρf - ρ_s)g\sinθ + (τ_B + μb)d_0]$. Then Eq. (32) becomes

$$\frac{dy}{dx} = Ay + B.$$  

By separation of variable, it follows $y = \frac{1}{A}[(e^{(x+C)}A + B)$, where C is the undetermined constant. From $y = 0$ as $x = 0$, it has $e^{xA} = -B$, thus $y = \frac{B}{A}(e^{Ax} - 1)$.

Qu. 11 3. Results and discussion: Can you explain how eqs. (38), (39) are derived (from (36), (37)) and how the un-numbered equations for the squared velocities of the solid and fluid are deduced.

Ans. The original manuscript: Eqs. (38) and (39) are derived from Eq. (33). This is hinted below Eq. (37) (“However, Eq. (33) provides the kinetic energy difference between two phases”). The un-numbered equations for the squared velocities of the solid and fluid are obtained from Eqs. (34), (36)-(39) and (35), (36)-(39), respectively.

Qu. 12 Introduction line 14: brush → bush
3 lines before 4 Conclusions: locale → location

Ans. We have replaced ‘brush’ and ‘locale’ by ‘bush’ and ‘location’, respectively.

References


2. Response to the second reviewer

R: Replace with,
I: Insert,
Ref.: Reference,
MC: Make clear,
IE: Improve English,
D: Remove.

Qu. 1 P2:
L10: and/or: R: and
L17: Himalaya-Karakorum
L10-26: Support with Ref.

Ans. We have changed ‘and/or’ into ‘and’ and added ‘Himalaya-Karakorum’ and the related references have been added. In page 26-30 in book “Debris flow: Mechanics, Prediction and Countermeasures”, author discussed the relationship between the density and the diameter of granules in a debris flow. It provides several examples to show this relationship between two items. The densities of the debris flow are in range of 1.5 – 2.0 t/m$^3$ and the solid concentration of 0.35-0.62 by volume. The average diameter of granules is about 0.01mm-10mm. By those discussion, we can found that a low-viscous debris flow with a density higher than 1400 kg m$^{-3}$ would contain a non-sediment fluid in which the diameter of granules is smaller than 0.05mm, whereas the high-viscous debris flow with a density higher than 1900 kg m$^{-3}$ would contain a non-sediment fluid in which the diameter of granules is smaller than 2mm.

Qu. 2 P3:
L3: two-phase fluid model: R: two-phase model;
L4-6: However, the two-phase fluid model describing a debris flow is still very difficult to explain via theoretical methods and to simulate accurately via numerical methods.: R: However, the two-phase models describing debris flows are still in development stages. Although, recently there have been substantial advances in simulating real two-phase debris flows [Pudasaini, 2012, 2014 (Acta Mech.)], construction of exact solutions are still very challenging [K. B. Khattri, 2014: ‘Sub-diffusive and Sub-advective Viscous Fluid Flows in Debris and Porous Media.’ M. Phil. Dissertation, Kathmandu University, School of Science, Kavre, Dhulikhel, Nepal, 2014.].
L7-9: To understand the dynamics of the debris flow, including its initiation, runout and deposition, finding out the velocity of the debris flow is important, which would be helpful to analyze and forecast the dynamics of the debris flow and then prevent its hazards.: MC!
L10: soils or rocks involved in a debris flow: R: soils or rocks, and fluid involved in a debris flow
L12: between the solid particles and the fluid: R: between the solid particles and the fluid [Pudasaini, 2012].
L16: I: “Pudasaini (2011) presented exact solutions for debris flow velocity for a fully
two-dimensional channel flows in which the velocity field through the flow depth and also
along the channel have been derived analytically.”
L16: Several models: R: Several other models
L22-23: Few theoretical results have been obtained to estimate the solid- and liquid-phase
velocities for a two-phase debris flow.: Which? Mention!
L24-28: IE.
Ans. P3:
L3: We have replaced ‘two-phase fluid model’ and ‘Pudasaini, 2005, 2012’ by ‘two-phase
model’ and ‘Pudasaini et al., 2005; Pudasaini, 2012’, respectively.
L4-6: We have changed ‘However, · · · via numerical methods.’ into ‘However, the two-
phase models describing debris flows are still in development stages. Although, recently
there have been substantial advances in simulating real two-phase debris flows (Pudasaini,
2012, 2014), construction of exact solutions are still very challenging (Khattri, 2014)’ and
L7-9: We have changed ‘To understand the dynamics of the debris flow, including its
initiation, runout and deposition, finding out the velocity of the debris flow is important,
which would be helpful to analyze and forecast the dynamics of the debris flow and then
prevent its hazards.’ into ‘To understand the dynamics of the debris flow, including its
initiation, runout and deposition, finding out the velocity of the debris flow is an important
factor for analyzing and forecasting the dynamics of the debris flow and then preventing
its hazards’.
L10: We have changed ‘soils or rocks involved in a debris flow’ into ‘soils or rocks, and
fluid involved in a debris flow’.
L12: We have changed ‘between the solid particles and the fluid’ into ‘between the solid
particles and the fluid (Pudasaini, 2012)’.
L16: We have added ‘Pudasaini (2011) presented exact solutions for debris flow velocity
for a fully two-dimensional channel flows in which the velocity field through the flow depth
and also along the channel have been derived analytically’.
L16: We have changed ‘Several models’ into ‘Several other models’.
L22-23: We have changed ‘Few theoretical results · · · debris flow’ into “Few theoretical
results have been obtained to estimate the solid- and liquid-phase velocities for a two-phase
debris flow (Chen et al., 2004; Chen et al., 2006)”.
L24-28: We have changed ‘Although some empirical formulas are introduced to calculate
the velocity of a debris flow at special location, such as the K631 debris flow that
occurred at the G217 highway (Tianshan highway) in Xinjiang Province and the Pingchuan
debris flow that occurred at the trunk highway from Xichang to Mul in Liangshan Yi
Autonomous Prefecture, Sichuan province (Chen et al., 2004, 2006). There is no general
formula to calculate the velocity of a debris flow’ into ‘Although some empirical formulas
are introduced to calculate the velocity of a debris flow at special location, such as the
K631 debris flow locating at the Tianshan highway in Xinjiang Province of China and the
Pingchuan debris flow locating at the trunk highway from Xichang City to Muli County in
Liangshan Yi Autonomous Prefecture, Sichuan Province, China (Chen et al., 2006).
Given that, there is no a general formula to calculate the velocity of a debris flow’.
Qu. 3 P4:
L1: the two-phase flow model: Which?
L6-9: Not clear how the obtained velocity would help estimating flow arrival time, and deposition area, etc. This can only be done by considering the full dynamical model and simulation that provides us with the temporal-spatial evolution of the flow depth and the velocities of the phases. So, improve writing!
L10-12: I do not fully agree, see comments above.
L20: I: “However, recently, by developing a general two-phase debris flow model, Pudasaini [2012] included several important physical aspects of the real two-phase debris mass flows with strong phase-interactions, including the generalized drag, virtual mass force, Newtonian, and solid particle concentration gradient enhanced non-Newtonian viscous stresses. These model equations have also been put in well structured and conservative form. Numerical simulations and possible applications of these models can be found in Pudasaini [2014], Pudasaini and Miller, 2012.”

Ans. P4:
L1: “the two-phase flow model” is formulas (3) and (4).
L6-12: We have changed ‘which would be useful for evaluating for evaluating the damage of a debris flow, estimating its arrival time, simulating its deposition area, predicting its risk, and so on’ into ‘which would be a useful factor for evaluating the damage of a debris flow, estimating its arrival time, simulating its deposition area, predicting its risk, and so on’.
L20: We have inserted ‘However, recently, by developing a general two-phase debris flow model, Pudasaini (2012) included several important physical aspects of the real two-phase debris mass flows with strong phase-interactions, including the generalized drag, virtual mass force, Newtonian, and solid particle concentration gradient enhanced non-Newtonian viscous stresses. These model equations have also been put in well structured and conservative form. Numerical simulations and possible applications of these models can be found in Pudasaini (2014), Pudasaini and Miller (2012a, b)’.

Qu. 4 P5:
L1-10: These three points can be written simply as [see, e.g., Pudasaini and Hutter (2007), Avalanche Dynamics, Springer, New York]:
1. One-dimensional, depth-averaged model (however, this contradicts with your statement in equations (18)-(22) where \( \frac{dv}{dy} \) is used!).
2. Finite mass.
3. Homogeneous and steady-state flow.
L11: Under the above assumptions and following the two-phase flow theory: R: Under the above assumptions and following the two-phase flow theory (see, e.g., Pudasaini, 2012 for more detail)
L16: Check the ‘dot’ operator (in equation (2)).
L22: In this study: R: For detailed model derivation, and how different types of forces and interactions can arises and should be introduced in a real two-phase mass flow model, we refer to Pudasaini (2012). However, In this study,
L22: Also mention the meaning of each term, variable, and parameter (e.g., \( v_s \), \( v_f \), \( \phi \), \( \rho_s \), \( \cdots \cdots \cdots \)) at place where they appear first. This will help the reader to follow the text.
1. In this study, the downstream direction is set as the $x$ direction, while the vertical direction to the channel bed is the $y$ direction (Fig. 1). We assume that the velocity along the $y$ direction is uniform, and thus the one-dimensional model for debris flow is mainly considered.

2. There are no external materials involved in the debris flow, and there is no transformation between the solid phase particles and liquid phase slurry. Three inner forces are involved in the model: the interactions among the solid phase particles, the interactions in liquid phase slurry and the interactions between the solid phase particles and liquid phase slurry.

3. A debris flow is assumed to be a homogeneous flow (Major and Iverson, 1999; Kaitna et al., 2007).

We have replaced ‘Under the above assumptions and following the two-phase flow theory’ by ‘Under the above assumptions and following the two-phase flow theory (see, e.g., Pudasaini, 2012 for more detail)’.

L22: For convenience, the parameters (e.g., $v_s$, $v_f$, $\phi$, $\rho_s$, $\cdots$) can be found in Table A1. Notation.

Qu. 5 P6:

L6: pressures are also the surface forces. So, either say surfaces for both or (better), say, viscous (shear) forces ($f$) and pressure forces ($P$), etc.

L5-25: These assumptions must be supported by physics of flow and references. - One of the major concerns in the MS is the definition of the pressure, which here is introduced as the impact pressure, which generally is a derived quantity but not a closure, or a rheological relation in fluid mechanics, mainly in the geophysical mass flows. Another problem with the definition (7) is the parameter $k$, which cannot be well constrained, but can only be a fit parameter. Further problem is that, the same parameter $k$ can not realistically model the fluid and solid (impact/dynamic) pressures. Moreover, in geophysical mass flows, the pressure, e.g., for solid is modelled as hydrostatic, and rate-independent relation (the solid normal load). So, pressures are field variables, but here these are used as derived quantities. This consistency and validity of these pressure definitions must be justified! The mixture density (8) and mixture velocity (11) are only defined but not used.

L25: after equation (12): I: “which is the buoyancy reduced normal load, see, e.g., Pitman and Le (2005), Pudasaini (2012).”

Ans. P6:

L6: We have revised it as ‘In order to estimate the velocities of a debris flow using Eqs. (5) and (6), the volume forces ($b_{sx}$ and $b_{tx}$) in a unit volume, pressures ($P_s$ and $P_t$), and surface forces ($f_{sx}$ and $f_{tx}$) in a unit volume beyond pressure (e.g., liquid resistance every phase, apparent mass force derived from acceleration and difference of velocity, and
interaction between particles, see, Chen, et al., 2006) firstly need to be given’. L5-25: These assumptions are supported by Chen, et al. (2011). The detailed description of the pressure can be seen in Chen, et al. (2006). The mixture density (8) and mixture velocity (11) are used in the computations regarding to $\bar{v}_2$ and $\bar{v}_3$ of Table 1.

L25: We have inserted ‘which is related to the buoyancy reduced normal load (see, e.g., Pitman and Le, 2005; Pudasaini, 2012)’ after equation (12).

Qu. 6 P7:
L4-9: IE, Ref.
L10-14: Provide Refs. for these definitions. It seems that these quantities are not consistent with dimensions!

Ans. P7:
L4-9: We have revised it as ‘In this study, for two-phase in a unit volume, the surface forces on control volume can been classified four by Chen et al. (2006). The surface forces of the solid phase $f_{sx}$ on control volume is divided into two parts: the traction of the liquid phase slurry outside control volume, $f_{sx1}$, and the force from the solid phase particles outside control volume, $f_{sx2}$. The surface forces of the liquid phase $f_{lx}$ on control volume is divided into two parts: the resistance from the solid phase particles outside control volume, denoted by $f_{lx1}$, and the resistance from the liquid phase slurry outside control volume, denoted by $f_{lx2}$.

L10-14: These definitions can been provided by Chen et al. (2006).

Qu. 7 P8:
L2: Bagnolds grain-inertial rheology is used to model the solid-granular-phase, which however, assumes more dilute collisional flows.

L7: $\lambda$: Ref. and provide expressions.
L9-10: should there be $\cos\alpha_i$.

L15: The fluid-phase assumes Bingham viscoplastic law.
L18: On the RHS: the second term should be with ‘+’!

Another major concern here is the use of the rheological equations and their validity! Bagnold and Bingham laws are used for the solid and the fluid, respectively. Now the questions is: Bagnold and Bingham relations are used to model the rheological behavior of the bulk mixture as a whole other than to model the solid and the fluid phases separately. Usually, the solid and fluid phases in real two-phase debris flow mixture are, respectively, modelled by applying the Coulomb-type frictional model and rate-dependent viscous flow model (Pudasaini, 2012). Which means that the use of the rheological models can be questioned. So, justify their use, and mention that: “However, a physically more meaningful and consistent would be the use of the Coulomb-type frictional model for the solid and the non-Newtonian viscous flow rheology for fluid as in Pudasaini (2012)”.

Ans. P8:
L2: Action of solid phase outside control volume to solid phase on control volume mainly involves impact effect between particles, the impact forces include dispersion force $P_0$ and and shear force $T_0$ between particles (see, Chien, 1989; Chen et al., 2006).

L7: $\lambda$ can been provided by Bagnold (1954) and $\lambda = 1 / \left[ (\alpha^0/\alpha)^{1/3} - 1 \right]$, where $\alpha^0$ is the maximum possible static volume fraction.
L9-10: Formulas of L9-10 can been provided by Chen et al. (2006).
L15: We have assumed the liquid-phase as Bingham viscoplastic material.
L18: On the RHS: the third term denotes the resistance of liquid phase slurry, thus the sign is ‘–’ (see, Chen et al., 2006; Chen et al., 2011, p. 46). The provided details of the rheological model can seen in Chen et al. (2004) and Chen et al. (2006).

Qu. 8 P9:
L3: and \( y \) is the internal depth of the debris flow body: this should have been introduced earlier!
L10-15: Mention that: “There are several model parameters in the proposed model including \( a, b, c, d_0, k \), etc. Constraining these parameters could be challenging. Such parameters, which could also be used as fit parameters, however, do not appear in a real two-phase debris flow model such as that presented by Pudasaini (2012)”.
L15-20: A principle question is that: pressures are included in (22) and (23). Then why do you need extra pressure terms (the last terms on the RHS of (5) and (6))? There is a redundancy!

Ans. P9:
L3: \( y \) is involved in local text of MS.
L10-15: Where appropriate, we have mentioned that: ‘There are several model parameters in the proposed model including \( a, b, c, d_0, k \), etc. Constraining these parameters could be challenging. Such parameters, which could also be used as fit parameters, however, do not appear in a real two-phase debris flow model such as that presented by Pudasaini (2012)’.
L15-20: Surface forces \( (f_s, f_f) \) in a unit volume beyond pressure is given on the above of formula (7) in the revised manuscript, e.g., liquid resistance every phase, apparent mass force derived from acceleration and difference of velocity, and interaction between particles (Chen, et al., 2006).

Qu. 9 P10:
L1-2: To simplify the calculation, the velocity of the solid phase in the \( y \) direction and the effect of turbulence in slurry are ignored.: This is not consistent, or at least not justified!
L3: \( d_0 \) is usually small enough: which value \( d_0 \) would take in practice? This is difficult, or not possible to say!
L6-12: As mentioned earlier, the last terms on the RHS of (26)-(27) are redundant! If not, explain!
P11-12:
Equations (34)-(35): The approach used in the model development and the physical correctness of final model equations must be justified and discussed! So, it would be better to mention here that: “Although the model solutions (34) and (35) providing the velocity estimates for the solid and fluid phases in a debris flow only utilize and retain the impact pressure difference between the solid and the fluid, and the Bingham viscoplastic parameter, they can only provide very basic qualitative picture of the solid and the fluid velocities. Also these solutions do not include any information about the volume of the debris material. Nevertheless, to develop velocity solutions for the solid and the fluid phases in a more consistent and physically more meaningful way, one must use a real and general two-phase debris mass flow model, such as the one developed by Pudasaini
(2012), that includes strong phase interactions through the generalized drag, virtual mass force, non-Newtonian enhanced viscous stress, and the evolving volume fraction of the solid-phase.”

There are some strange effects: E.g., for $\phi = 0.5$ (which may be a possible scenario), the second terms on the RHS of (34) and (35) associated with fluid disappears! Check, and discuss it!

**P12-13:**
L5(P12)-L13(P13): D: Because this does not add anything in the analysis! Also, there are inconsistency in the descriptions of, e.g., $M_2$, because with increasing $\tau_B$ and $\mu$ the KE must decrease, but here it increases!

**Ans. P10:**
L1-3: We have revised it as ‘If the effect of turbulence in the liquid slurry is not considered, then Eq. (21) can be simplified as

$$f_{tx2} = a\mu d_0^2 + (\tau_B + \mu)b_0.$$  

(22)

Further, if the velocity of the liquid phase slurry with respect to $y$ submit to linear function, i.e. $a = 0$, then Eq. (22) can be simplified as

$$f_{tx2} = (\tau_B + \mu)b_0.$$  

(23)

Combining Eqs. (16) and (18) yields

$$f_{sx} = f_{sx1} + f_{sx2} = 3k\varphi \left( \rho_f v_{tx2}^2 - \rho_s v_{sx2}^2 \right) + \int_0^{d_0} 0.041\rho_s (\lambda d_0)^2 \left( \frac{dv_{sy}}{dy} \right)^2 dy.$$  

(24)

Combining Eqs. (17) and (21) yields

$$f_{tx} = f_{tx1} + f_{tx2} = -\frac{3k\varphi}{2d_e} \left( \rho_f v_{tx2}^2 - \rho_s v_{sx2}^2 \right)$$

$$+ \frac{4a\rho_f v_{tx2}^2 d_0^3}{5} + ab\rho_f^2 \eta^2 d_0^4 + \frac{\rho_f b^2 \eta^2 d_0^3}{3} + a\mu d_0^2 + (\tau_B + \mu)b_0.$$  

(25)

Next, we will take steady flow of debris flow (Chen, 1988; Chen et al., 2004; Jan and Shen, 1997) and linear distribution of velocity of the liquid phase slurry with respect to $y$ (Chen et al., 2006) as an example. Then Eq. (25) can be written as

$$f_{tx} = -\frac{3k\varphi}{2d_e} \left( \rho_f v_{tx2}^2 - \rho_s v_{sx2}^2 \right) + (\tau_B + \mu)b_0.$$  

(26)

To simplify the calculation, the velocity variation of the solid phase particles along depth of debris flow body is omitted (Chen et al., 2006), then Eq. (24) can be taken the form

$$f_{sx} = \frac{3k\varphi}{2d_e} \left( \rho_f v_{tx2}^2 - \rho_s v_{sx2}^2 \right).$$  

(27)

L6-12: The last terms on the RHS of (26)-(27) are not redundant. They are the important terms and can be explain in the response of Qu. 5.
Equations (34)-(35): The approach can be verified by two real-world debris flows (Sect. 3. Results and discussion) and we have mentioned that: ‘Although the model solutions (34) and (35) providing the velocity estimates for the solid and fluid phases in a debris flow only utilize and retain the impact pressure difference between the solid and the fluid, and the Bingham viscoplastic parameter, they can only provide very basic qualitative picture of the solid and the fluid velocities. Also these solutions do not include any information about the volume of the debris material. Nevertheless, to develop velocity solutions for the solid and the fluid phases in a more consistent and physically more meaningful way, one must use a real and general two-phase debris mass flow model, such as the one developed by Pudasaini (2012), that includes strong phase interactions through the generalized drag, virtual mass force, non-Newtonian enhanced viscous stress, and the evolving volume fraction of the solid-phase’. It is correct. \( \phi = 0.5 \) is a strange value. In this case, the effect of \( \rho_f \) is disappeared in the first term for the RHS of Eq. (35). However, it is also appear in the second term of the equation. So the term \( \rho_f \) also affect the velocity of \( v_{fx} \). However, it is not very clear what happens for the debris-model under this special values and it is also difficult to explain it clearly. We will try to figure out it in the future work.

L5(P12)-L13(P13): To facilitate understanding, we give some descriptions and definitions for Eqs. (34) and (35) in the original manuscript. \( M_2 \) is only to define an expression.

**Qu. 10**

P13:

L14-25: IE. Improve the discussion with mechanically more appropriate statements. E.g., as the equivalent diameter of solid particles increases, the solid-phase velocity of a debris flow decreases very slowly whereas the liquid-phase velocity increases very slowly: ‘the liquid-phase velocity increases very slowly’; this is not consistent with the physics of flow! Otherwise provide data! So discuss and mention that: “Such discrepancies may have been emerged do to the very simplified model consideration, or some possible inconsistencies in the use of the rheological models considered here. These problems could have been avoided by using more complete and real two-phase debris flow model (Pudasaini, 2012) which includes strong phase interactions.”

**Ans. P13:**

L14-25: Thanks. We have changed the discussion as Reviewer’s suggestion.

**Qu. 11**

P14:

L2: the K631 debris: what it is?: Ref.

L5-11: These comparisons are not so meaningful. Because: you must compare (with respect to the involved):
- the flow volume,
- travel distance, etc.

Discuss on this!

Again, you have not proven how ‘the estimation method for the velocities of a debris flow can be widely used for a real-world debris flow’! You used very strong statement with relatively weak results. Otherwise, justify!

**Ans. P14:**
L2: The K631 debris flow locating at the Tianshan highway in Xinjiang Province of China can be described in introduction (Chen et al., 2004; Chen et al., 2006).

L5-11: We prove the validity of our results. It is better than Chen’s results (2006). The flow volume and travel distance are considered for follow-up studies. We have revised ‘the estimation method for the velocities of a debris flow can be widely used for a real-world debris flow’ as ‘the estimation method for the velocities of a debris flow can be effectively used for a real-world debris flow’.

Tables:

Qu. 12 P19:

- equivalent radius of control volume: $R$: equivalent height or length of control volume
- Also improve other items.

Ans. P19:

- We have replaced ‘equivalent radius of control volume’ by ‘equivalent height of control volume’. The other items are also revised accordingly.

Figures:

Qu. 13 P20:

- Caption: R: Debris flow configuration, and definition of variables and parameters.
- Also, make a debris flow profile and include the equivalent volume in it!

Ans. P20:

- Thanks. We have added another Figs. (a), (b) as Reviewer’s suggestion.

Qu. 14 P21:

- Mention (in the main text) that: “Such exact solutions have also been presented previously by Pudasaini (2011) for avalanche and debris flows.”
- Fig. 2: Caption: put parameter’s dimensions. $\rho_f = 1500$ is too big, $\phi = 0.1$ is too small, $(\tau_B + \mu b)d_0 = 100$: explain why this value is chosen.
- As you explained in the text, $d_e$ must be larger than 0.2 m. This also applies to other figures!
- At the model solution says, at $x = 0$, both $v_s$ and $v_f$ must be zero, but here they are not! Check this!
- To check the model performances, results must also be plotted for more dense flows (i.e., for $\phi = 0.65$).
- For which volume these results are plotted. It seems that your model solution does not include this information. This is a problem here!
- Mention in the main text that: “For such a large velocity difference, at least the drag and the virtual mass force must have been included in the model as in Pitman and Le (2005) and Pudasaini (2012). However, here the model does not consider such effects.”

Ans. P21:

- We have mentioned that: “Such exact solutions have also been presented previously by Pudasaini (2011) for avalanche and debris flows” in main text.
- Fig. 2: Caption: the parameter’s dimensions: $\rho_f$, $\phi$ and $(\tau_B + \mu b)d_0$ are according to Chen et al. (2006)
- Following Chien (1989), particles which diameter is less than 0.1 m in viscous debris flow often form mass and move at certain direction with the same velocity, while particles that
diameter is over 0.1 m move at jumping in debris flow gully. However diameter of particle at suspension state in thin debris flow is less than 0.02 m. Therefore, generally, taking particle whose diameter is less than 0.02 m as equivalent slurry, the other belongs to solid (Chen et al. 2006).

- At $x = 0$, both $v_s$ and $v_f$ must be zero, but here they are not! The existence of this case refers to the drawing error at $x = 0$ by using Matlab software.

- Here, we take some parameters according to the results of Chen et al. (2006). The method is also suitable for both viscous debris flow and thin debris flow. We have Mentioned in the main text that: ‘For such a large velocity difference, at least the drag and the virtual mass force must have been included in the model as in Pitman and Le (2005) and Pudasaini (2012). However, here the model does not consider such effects.’

**Qu. 15 P22:**

Fig. 3: Caption: Mention in the text that: “However, 10% increase in the solid volume fraction resulted only in very slight decrease in the solid and fluid velocities.”

**Ans. P22:**

Fig. 3: We have mentioned in the corresponding text that: ‘However, 10% increase in the solid volume fraction resulted only in very slight decrease in the solid and fluid velocities.’

**Qu. 16 P23:**

Fig. 4: Caption: Mention in the text that: “However, 10% increase in the particle diameter (as a parameter) resulted in almost no change in the solid and fluid velocities.”

**Ans. P23:**

Fig. 4: We have mentioned in the corresponding text that: ‘However, 10% increase in the equivalent diameters of solid particles resulted in almost no change in the solid and fluid velocities.’

**References**


We would highly appreciate if you could take necessary action.

Looking forward to hearing from you.

Best wishes to you!

Yours sincerely,

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Zuohuan Zheng
Yang Gao