

Dear Referee,

Thanks for your constructive comments. We have tried our best to rebuttal your comments line by line as following:

**(1)Referee: WHY the need to go to such a system; simply saying that 3 degrees of freedom is necessary for chaos is superficial.**

Authors: Based on the experimental observations, Ruina (1980, 1983), Ruina and Rice (1983) have proposed the following empirical relation for rock friction

$$\tau = \tau^* + A \ln(v/v^*) + B \ln(v^* \theta_i / L_i).$$

The state variables ' $\theta_i$ ' are defined using the slip law of friction as

$$\frac{d\theta_i}{dt} = -\frac{v\theta_i}{L_i} \ln\left(\frac{v\theta_i}{L_i}\right), \text{ where } i = 1, 2, 3$$

The above model with one state variable based RSF model(1sRSF) is generally used to explain stick-slip behaviour but this model doesn't explain the chaotic behaviour of the sliding system. However, the RSF law with two state variables i.e., 2sRSF does show the chaos (Gu et al., 1984; Gu and Wong, 1994; Niu and Chen, 1994, 1994; Becker, 2000). As a result, a natural question arises how the non linear behaviour of the RSF model changes if one more state variable is added in the friction model that is, the RSF model with three state variables (3sRSF). This is what we have analyzed in this paper without paying much attention on the practical/physical significance of the 3sRSF model. In recent times, the RSF model with two state variables (2sRSF) are widely being used to validate the sliding friction of rocks at higher temperatures by justifying the multiple mechanisms of friction becomes active in such a scenario (King and Marone, 2012; Lui, 2007). We believe that the 3sRSF model could be promising in high sliding and temperature experiments on rock surfaces.

**(2)Referee: Are there any PHYSICAL features of the underlying problem that the new models (including their 2 state models) describe that the original 1980s models cannot.**

Authors: It is to be mentioned that the original RSF model (1980s) was proposed with one state variable (1sRSF) model. However Ruina (1980, 1983) argued that 1sRSF model is not sufficient to explain friction experiments on rock surfaces. He used the 2sRSF model to fit the experimental data and also justified the need of 2sRSF. Gu et al. (1984) studied the 2sRSF law numerically and predicted its chaotic behaviour. In recent times, King and

Marone (2012) have reported that the 2sRSF explains better the experimental data pertaining to high temperature sliding than the 1sRSF model. They have attributed the reason to the possible onset of a second mechanism of friction at higher temperatures.

**(3)Referee: There is no connection with the underlying physics presented here, and the authors simply use some canned programs to calculate Lyapunov exponents (why do they get more than one; that does not conform with the traditional definition for the Lyapunov exponent) and obtain a fractal dimension of 5.7 (why, when dealing with a 3 degree of freedom system).**

Authors: Addition of one more state variable in the 2sRSF is believed to explain more complex form of friction at rock surfaces, for instance at high temperature and sliding velocities. Moreover, the number of Lyapunov exponents (LEs) in a dynamical system is generally equal to the number of the degree of freedom (dimensions) of the system. But the largest value of the LE is generally reported. As Niu and Chen (1994), Becker (2000) have reported all the Lyapunov exponents in the published paper. Following them, we have also presented all the Lyapunov exponents in the present paper. Moreover, calculation of the fractal dimension of the present dynamical system, which is equal to 5.7, is not correct at all. Since all the Lyapunov exponents are positive thus the formula for calculating fractal dimension is not valid. As an error, we shall remove this estimation from the present article.

**(4)Referee: it is not sufficient for them to apply these tools to data generated from their new model without asking what kind of outcomes emerge from the original 1980s models.**

Authors: The original 1980s RSF models explained well stiffness dependence of stick-slip motion of hard surfaces. The original 1980s RSF models also include two state variables (2sRSF) but the chaotic behaviour of the 2sRSF was not studied in detail in those times. In 1990s, Gu and Wong (1994), Niu and Chen (1994,1995), Becker (2000) studied the non-linear behaviour of the 2sRSF model in detail using non-linear dynamical tools such as Poincare map, Bifurcation diagram, Lyapunov exponents etc. These studies firmly established the route of chaos as “Period doubling”. Further, Niu and Chen (1995), Becker(2000) reported the fractal dimension of 2sRSF is equal to 2.11, and have also reported the Feigenbaum number which nearly converges to universal Feigenbaum constant (4.669201).

**(5)Referee: does this model given any added physical insight into the frictional problem that motivated the original studies?**

Authors: Yes, the present friction model has revealed that all the Lyapunov exponents are positive. This observation is, in contrast, with the 2sRSF model which shows one positive, one negative and one nearly equal to zero (Niu and Chen, 1995, Becker, 2000). Moreover, despite being the same route of chaos that is “period doubling”, the 3sRSF system shows the chaotic behaviour after four periods while the 2sRSF results in chaos after sixteen periods. The reason is obvious due to number of positive LEs in these two systems. Further linear stability analysis shows that the critical stiffness of the sliding system predicted by the 3sRSF model is larger than the corresponding 2sRSF model. Accordingly, stiffness at which chaos occurs in the 3sRSF is also larger than the 2sRSF law ( Niu and Chen,1994). Further, stress drop during the chaos with the 3sRSF model is larger than the corresponding 2sRSF model. On the basis of these observations, we establish that the 3sRSF is more chaotic than the 2sRSF.

**(6)Referee: Just because their new model is more complicated does not of itself justify its publication unless compelling reasons emerging from the Physical problem of frictional slip etc. as well as a clear clarification of the meaning of their MatLab results is provided.**

Authors: Recent friction sliding experiments at high temperature and sliding velocities have shown more complex form of friction. So the more complex form of the RSF model could be justified. Of course, further study needed to justify the addition of third state variables in the RSF model. Moreover, the physical meaning of MATLAB plots are explained as

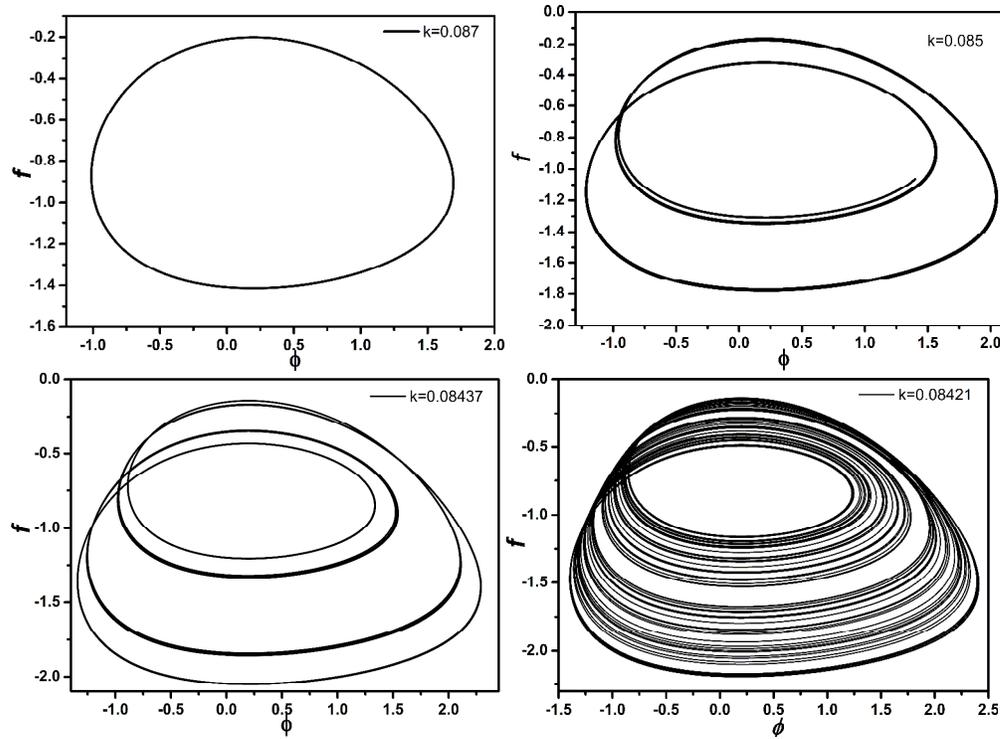


Fig.3-6: Phase diagram  $f$  vs.  $\phi$  for  $k = 0.087$ ,  $k = 0.085$ ,  $k = 0.08437$  and  $k = 0.08421$   $\beta_1 = 1.0$ ,  $\beta_2 = 0.84$ ,  $\beta_3 = 0.38$ ,  $\rho = 0.048$  and  $\rho_1 = 0.034$  for initial condition  $[0,0,0,0]$ .

The plots in Figs.3-6 of the paper show the periodic behaviour with fixed oscillating amplitude of the stress level for spring stiffness  $k = 0.087$ . As stiffness of the connecting spring reduces further to  $k = 0.085$ , period doubling behaviour is seen and now amplitude of stress level fluctuates between two stages of vibration. However the stress build-up and drop mechanism is still in a periodic manner. Upon decreasing the stiffness of the sliding system to  $k = 0.08437$ , its behaviour goes to period quadrupling in which amplitude of stress fluctuates at four stages of vibration thus the slip surface is more prone to fail. Finally the sliding system goes to chaotic behaviour at  $k = 0.08421$  and now build-up and sudden-drop of stress amplitude is irregular and larger in comparison to the period doubling or quadrupling etc. thereby more chances of earthquake nucleation. These results are in confirmation with the Bifurcation diagram in Fig.7 of the manuscript.