Interactive comment on “Linear and Non-linear Stability Analysis of the Rate and State Friction Model with Three State Variables” by N. Sinha and A. K. Singh

Anonymous Referee #2

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In this paper, the authors have analyzed the quasi-static frictional sliding of a spring-block slider system under the Rate-State Friction (RSF) formalism with three state variables. The study of 3-state RSF systems have not been carried out in the literature, the authors claim that this warrants the study of mechanical behavior of such systems. I feel that that this is not justifiable scientific motivation. There is good reason why 3-state variable systems have not been studied in the literature, laboratory friction experiments simply do not seem to suggest the need for this additional complexity. While some large rate-step experiments seem to point towards a second state variable (Marone, 1998; Ruina 1983) even this, in itself, is by no means a robust feature of experimental rate-stepping data. I find it difficult to understand the need for adding yet
another, by all accounts unnecessary, degree of freedom to the system.

Besides this clear motivational short-coming, the paper suffers from a lack of proper discussion of the background of the RSF formalism. For example, in Eq. 1 (which misses the summation sign on $\theta_i$), the authors neglect to mention that they are using the Slip law for their simulations. Further, there is no discussion of why they choose the Slip law for their simulations. There is also no discussion of how these existing formulations of RSF fail to explain observed experimental data. There needs to be some discussion of how sensitive their results are to the choice of the state evolution law i.e. 3 Slip law state variables versus 3 Aging law state variables. One would expect the critical stiffness estimate to be insensitive to this choice (given Aging and Slip laws are asymptotically identical near steady state) but behavior under large perturbations from steady state is likely to be sensitive to this choice.

Also, given that a considerable amount of work has been done on evaluating the critical stiffness for the 1- and 2-state variable state evolution equations (Gu et al., 1984, Becker, 2000), the authors need to discuss explicitly how their estimate of the critical stiffness relates to the known expressions of the critical stiffness for these simpler formulations of state evolution. In a related point, the authors make the following claim in the abstract: “Linear stability analysis shows that critical stiffness, at which dynamical behaviour of the sliding system changes, increases with number of state variables”- it is likely that this conclusion is not generally true. This paper would definitely benefit from a section which systematically studies how the critical stiffness varies as a function of $A, B_1, B_2, B_3, L_1, L_2$ and $L_3$.

There are other technical problems with the paper:

Pg. 2, L6: It is incorrect that a second state variable was required to explain chaos in frictional slip. The second state variable is used to explain experimental observations of slip-weakening in response to rate steps.

Pg. 3, L4, L10: The conditions on the Lyapunov exponents for hyper-chaos as stated
are confusing. Does the sum of all LE’s require to be +ve or -ve? The authors seem to suggest both at some point.

Pg. 12, L9: How is the fractal dimension 5.7 for a 4D (3 states, 1 slip rate) system?

The writing style, grammar needs to substantially improve throughout for this paper to be publishable in an international journal. Copy-editing by a native English speaker might be necessary.

Overall, given that the entire exercise carried out by the authors has very little scientific motivation based on laboratory friction experiments, I find it hard to recognize what exactly would the field of friction constitutive equations stand to gain from this work.