Dr. Antonio Paz González  
Editor Special Issue “Multifractal Analysis in Soil Systems”  
Nonlinear Processes in Geophysics  

Subject: Submission of revised manuscript to *Nonlinear Processes in Geophysics*

Dear Editor,

On behalf of all authors, I would like to submit the following manuscript entitled: “Multifractal behaviour of the soil water content of a vineyard in NW Spain during two growing seasons”, by José Manuel Mirás-Avalos, Emiliano Trigo-Córdoba, Rosane da Silva-Dias, Irene Varela-Vila, and Aitor García-Tomillo.

We thank all the comments and suggestions posed by the reviewers. All comments were taken into account and changes according to them were performed throughout the text. A specific answer to all the reviewers’ comments is provided at the end of this letter. Besides, we moved a table and two figures to supplementary materials in order to alleviate manuscript length. We also corrected several mistakes in the reference list.

We hope that, at its present form, our manuscript would reach the high quality standards for publication on *Nonlinear Processes in Geophysics*.

Looking forward to hearing from you.  
Sincerely yours,

*José Manuel Mirás Avalos*  
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Reviewer: 1 (M.G. Wilson)

This paper is precise and concrete. The paper describes dynamics of soil water content and considers the multifractality of these time data series of a vineyard in NW Spain. The paper is showing scaling properties of soil water content time series, these properties were different for the rain-fed and irrigation use, and higher heterogeneity under irrigation conditions

*Thank you very much for your comment, Dr. Wilson.*
The aim of this paper is to describe soil water dynamics in vineyard and to assess multifractality. This topic is very interesting, because there is no application of multifractality to the temporal evolution of the soil water content in vineyard under two different water supply methods. This paper presents on an interesting subject and well written. The multifractality is analysed using the classical method of moments. A question will be if the authors have detected the effects of seasonal trend on the multifractal analysis.

Thank you very much for your comments. Regarding your question about the effects of seasonal trend on the multifractal analysis, we are not sure about what you mean. If you refer to differences in multifractal parameters from one season to the other; we addressed this on the former version by stating that “2012 data series presented a higher heterogeneity than those from 2011” (page 8, lines 13-14). In the current version of the manuscript we added “Moreover, the width of the Dq spectra increased from 2011 to 2012 in both treatments, mainly in the 20 and 40 cm depths” (page 8, lines 27-28). We also added “Our results showed that the width of the singularity spectra increased in both treatments from 2011 to 2012 (Table 2)” (page 9, lines 26-27). Finally, in the conclusions section, we added “, which tended to increase in the second year of the study (2012)” (page 10, lines 21-22). We hope that these additions answer your question.

Specific and technical comments

Page 3 line 18 when calculating the soil water budget would have to take into account both the interception, and surface storage, and consider surface runoff.

Yes, you are right, but we did not consider surface runoff because it was negligible on the studied plot. Surface storage and water interception by the canopy were not accounted for because our objective was not calculate a precise soil water budget but provide irrigation according to midday stem water potential readings and crop evapotranspiration, as described in this portion of the manuscript.

Page Line 16 “0.4 mg of suspended soils”, units must be mg/l or other concentration units.

Yes, you are right, it is mg/L. We corrected it in the revised version of the manuscript.
Page 4 line 8 “...the equation provided by the manufacturer was used for transforming permittivity data registered by the probes into soil water content” justify this statement, because FDR calibration strongly depends on soil type.

Yes, you are right, soil type greatly influences the FDR probe readings. However, we used the equation provided by the manufacturer because we did not use these measurements for scheduling irrigation since we relied on stem water potential readings. Besides, the default equation can be used for relative or differential measurements since we are comparing the performance of two probes in the same soil but under two different irrigation regimes. We included a brief text explaining this and providing a citation from a work by Paraskovas et al. (2012) in International Agrophysics in order to justify our procedure.

Page 7 line 15, add “and soil evaporation”

Added as suggested.

Page 7. Line17 “...Indeed, our results suggest that the water amount applied through irrigation was enough for fulfilling vineyard water requirements over the two growing seasons studied” justify this statement Table 1 Use ETc data, I don’t know if the depth of irrigation dose is net or gross. The dose of irrigation calculated seems very low, because if we calculate the approximate dose irrigation as precipitation less than 50% of crop evapotranspiration assuming negligible value interception, surface storage and surface runoff, the value obtained would be much higher than the value shown in table

We removed the sentence because we did not provide data on grapevine vegetative growth, yield and berry quality, which were slightly affected by irrigation and made us to conclude that. These data are pending for publication and have been submitted to another journal; therefore, we considered removing this sentence as the best option since it does not affect the main focus and conclusions from the current manuscript.

We agree with you in the sense that Table 1 led to confusion since the data reported there referred to the whole period of measurements and not exclusively to the irrigation period. Besides, this table provided similar information to that displayed in figure 2. Therefore, we removed the table from the main manuscript and moved it to supplementary materials and added the information referred to the irrigation period for each year.

Fig 3 year 2011 around day 230 there is increase in rain-fed treatment in 20 cm depth but this increase is not showed in irrigated plot.

This is true; it was caused by three rainfall events happening on consecutive days from the day 234 to the day 236, accumulating 15 mm. The 20 cm depth sensor in the rain-fed treatment responded quickly to this increase (increases in soil water content were observed in day 235), whereas the
sensor in the irrigated treatment responded more slowly (day 237). Moreover, when checking the data, we observed that this increase in the soil water content at 20 cm depth in the rain-fed treatment lifted the values up to those observed in the irrigated treatment at the same depth. Therefore, it seemed to be just a delay in the response of the sensor since the water content at 20 cm depth in the irrigated treatment also increased (in a lower proportion than that in the rain-fed treatment) after two days of the last rainfall event.

Fig 5 and 7. Improve figure quality

Thank you, quality of the figures has been improved as you suggest. However, in order to reduce manuscript length, we moved figure 5 to the supplementary material.
Multifractal behaviour of the soil water content of a
vineyard in NW Spain during two growing seasons

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Abstract

Soil processes are characterized by a great degree of heterogeneity, which may be assessed by
scaling properties. The aims of the current study were to describe the dynamics of soil water
content at three depths in a vineyard under rain-fed and irrigation conditions and to assess the
multifractality of these time data series. Frequency domain reflectometry (FDR) sensors were
used for automatically monitoring soil water content in a vineyard located in Leiro (Ourense,
NW Spain). Data were registered at 30-minute intervals at three depths (20, 40 and 60 cm)
between 14th June and 26th August 2011 and 2012. Two treatments were considered: rain-fed
and irrigation to 50% crop evapotranspiration. Soil water content data series obeyed power
laws and tended to behave as multifractals. Values for entropy ($D_1$) and correlation ($D_2$)
dimensions were lower in the series from the irrigation treatment. The Hölder exponent of
order zero ($\alpha_0$) was similar between treatments; however, the widths of the singularity
spectra, $f(\alpha)$, were greater under irrigation conditions. Multifractality indices slightly
decreased with depth. These results suggest that singularity and Rényi spectra were useful for
characterizing the time variability of soil water content, distinguishing patterns among series
registered under rain-fed and irrigation treatments.
1 Introduction

Soil water storage variability is strongly related with topographical, geological, edaphic and vegetation factors (Braud et al., 1995). These environmental factors and processes (rainfall, evapotranspiration, runoff) do not operate independently but as a conjunction of processes with nested and complex effects. Overall, this results in a distribution of soil water storage that varies as a function of the temporal and spatial scales. Therefore, similar to other soil properties and processes (Western and Blöschl, 1999; Zeleke and Si, 2006), soil water storage along time is a complex process characterized by a lack of homogeneity; heterogeneity in space and/or time is a feature that can be described by scaling procedures.

Fractals have been widely employed in soil science, as soil properties may be described through scale invariance concepts (Tyler and Wheatcraft, 1990; Perfect et al., 1996; Vidal Vázquez et al., 2007; Biswas et al., 2012a). More recently, several authors performed multifractal studies of heterogeneous time data series. For instance, Jiménez-Hornero et al. (2010) described ozone time series using the multifractal formalism. Rodríguez-Gómez et al. (2013) used a multifractal approach for characterizing solar radiation time series.

Soil water content can be automatically estimated by using sensors that measure variations in the soil dielectric constant, since it is strongly related with soil water content (Mestas-Valero et al., 2012). This parameter is characterized by its spiky dynamics, with sudden and intense peaks of high frequency activity, mostly at soil surface. Several studies have described scaling patterns for the behaviour of soil water content spatial distribution (e.g. Kim and Barros, 2002; Biswas et al., 2012b); however, multifractal analyses of continuously measured soil water content are scarce, except for a study on rain-fed grassland (Mestas-Valero et al., 2011). Therefore, the aim of the current work was to describe soil water dynamics in a vineyard subjected to two different treatments (rain-fed and irrigated) and to assess multifractality of these data series over two consecutive seasons.
2 Materials and Methods

2.1 Description of the study area

The experiment was conducted over two consecutive growing seasons (2011-2012) in a 0.2-ha vineyard (*Vitis vinifera* L.) planted with cultivar ‘Albariño’, located in the experimental farm of the Estación de Viticultura e Enoloxía de Galicia (EVEGA), in Leiro (42° 21.6’ N, 8° 7.02’ W, elevation 115 m), Ourense, Spain (Fig. 1). Vines were grafted in 1998 on 196-17C rootstock and trained to a vertical trellis on a single cordon system (10-12 buds per vine). Rows were east-west oriented, spacings between vines and between rows were 1.25 and 2.4 m, respectively (3333 vines ha⁻¹). The soil at the site was sandy-textured (64% sand, 16% silt, 20% clay), slightly acidic (pH 6.3), medium fertility (2.7% organic matter) and with a rather shallow profile (≈1.2 m). The climate of the studied site is temperate, humid with cool nights (Fraga et al., 2014).

2.2 Experimental design

The reference evapotranspiration (*ET*₀) per week for the site was calculated from weather variables recorded at a station located 150 m away from the experimental vineyard using the Penman-Monteith equation (Allen et al., 1998). The *ET*₀ was then used, along with a constant crop coefficient (*Kₑ* = 0.8) to compute the amount of water required by the vines (Trigo-Córdoba et al., 2015). Precipitation was substracted from *ETₑ* each week. The calculated amount of water was applied the following week.

Treatments consisted of a rain-fed control and an irrigation to the 50% of *ETₑ*. Irrigation was applied from late June – early July (after bloom) till mid-August, approximately two weeks prior to harvest through two pressure-compensated emitters of 4 L h⁻¹ located 25 cm on either side of the vine. Irrigation water was of good quality, with pH of 6.35, electrical conductivity of 163.4 μS cm⁻¹ and 0.4 mg L⁻¹ of suspended solids. The water amount applied each season was 40 and 50 mm for 2011 and 2012, respectively (Supplementary Table S.1).

2.3 Measurements

The volumetric soil water content was continuously monitored through the soil profile in two spots of the experimental vineyard (one in the rain-fed treatment and another in the irrigated treatment) using two capacitance probes (EnviroSCAN, Sentek, Australia), based on the
frequency domain reflectometry (FDR) technique. Each probe was equipped with three sensors installed on an access tube at 20, 40 and 60 cm depth and connected to a datalogger. The probes were properly maintained for recording soil water content at half-hour intervals over the 2011 and 2012 seasons. Here, data from the irrigation period (mid-June to late-August) are reported.

In each treatment, the probe was located within two vines (Fig. 1), avoiding to be close to the emitters (25 cm from the emitter and 50 cm from the vine trunk, approximately). The equation provided by the manufacturer was used for transforming permittivity data registered by the probes into soil water content since we only wanted to compare relative contents between these two irrigation regimes. Previous work suggest that soil type greatly affects the FDR readings, but the default equation is valid for differential measurements (Paraskovas et al., 2012).

2.4 Multifractal analysis

The concepts of multifractals and their estimation methods that were used in the current study are next summarized. For detailed descriptions about multifractals, further information can be found in Chhabra et al. (1989) and Everstz and Mandelbrot (1992).

To implement the multifractal analysis of one-dimensional soil water content time distributions supported on a given interval \( I = [a, b] \), a set of not-overlapping sub-intervals of \( I \) with equal length is required. A common choice is to consider dyadic scaling down (Everstz and Mandelbrot, 1992; Caniego et al., 2005), which means successive partitions of \( I \) in \( k \) stages (\( k = 1, 2, 3… \)). Hence, at each scale, \( d \), a number of segments, \( N(\delta) = 2^k \) are obtained with characteristic time resolution, \( \delta = L \times 2^{-k} \), covering the whole extent of \( I \).

Multifractal approach applied to time series has already been described (Jiménez-Hornero et al., 2010), hence, we only summarize the technique used in the current study. The time interval of soil water content data series, \( L \), varied from half an hour to two months and the minimum time resolution, \( \delta_{\text{min}} \), was chosen accounting for containing at least one half-hourly averaged soil moisture data, \( \theta_{\text{min}} \), at every initial interval. According to this, the probability mass distribution, \( p_i(\delta) \), at time resolution \( \delta \) was estimated as:

\[
p_i(\delta) = \frac{\theta_i(\delta)}{\sum_j (\theta_{\text{min}})_j}
\]
where $\theta_i$ is the water content of the $i^{th}$ interval and $n_{ini}$ is the number of initial intervals with mean soil water content $\theta_{ini}$.

The method of the moments was used (Chhabra et al., 1989) to analyze the multifractal spectrum of the probability mass function, $p_i(\delta)$. The partition function $\chi(q, \delta)$ was estimated:

$$\chi(q, \delta) = \sum_{i=1}^n p_i(\delta)^q$$

were moment $q$ is a real number between $-\infty$ and $+\infty$.

A log-log plot of the partition function versus $\delta$ for different values of $q$ yields:

$$\chi(q, \delta) \propto \delta^{-\tau(q)}$$

were $\tau(q)$ is the mass scaling function of order $q$. The functions $f(\alpha)$ and $\alpha$ can be obtained by Legendre transformation of the mass exponent, $\tau(q)$, as: $f(\alpha) = \alpha(q) - \tau(q)$ and $\alpha(q) = d\tau(q)/dq$, respectively. Log-log plots of $\chi(q, \delta)$, versus $\delta$, however, typically exhibit linearity across a limited scale range (e.g. Posadas et al., 2003), which results in drawbacks when using the moment method to obtain the singularity spectrum.

The direct method (Chhabra and Jensen, 1989) avoids inaccuracies associated to the estimation of $\alpha(q)$ by Legendre transformation. This method is based on the calculation of the contributions of individual segments, $\mu_i(q, \delta)$, to the partition function, which are defined as:

$$\mu_i(q, \delta) = \mu_i^q(\delta) / \sum_{i=1}^{N(\delta)} \mu_i^q(\delta)$$

Then, using a set of real numbers, $q$, ($-\infty < q < -\infty$), the relationships applied to calculate $f(\alpha)$ and $\alpha$, can be expressed as:

$$f(\alpha(q)) \propto \sum_{i=1}^{N(\delta)} \mu_i(q, \delta) \log[\mu_i(q, \delta)] / \log(\delta)$$

and

$$\alpha(q) \propto \sum_{i=1}^{N(\delta)} \mu_i(q, \delta) \log[\mu_i(q, \delta)] / \log(\delta)$$

The $f(\alpha)$–$\alpha$ spectrum is reduced to a point for monofractal scaling type. The minimum scaling exponent ($\alpha_{min}$) corresponds to the most concentrated region of the measure, and the maximum exponent ($\alpha_{max}$) corresponds to the rarefied regions of the measure. A plot of $f(\alpha)$ vs. $\alpha$ is called multifractal spectrum. It is a downward function with a maximum at $q = 0$. The
width of the multifractal spectrum \((w = \alpha_{\text{max}} - \alpha_{\text{min}})\) indicates overall variability (Moreno et al., 2008) similar to the nugget effects in geostatistics. For each data series, we calculated multifractal spectrum with \(q\) from \(-10\) to \(+10\) in steps of \(0.5\), fine enough to show the multifractal behaviour in the studied moment range.

Multifractal measures can also be characterized on the basis of the generalized dimension, \(D_q\), of the moment of order \(q\) of a distribution, defined by Grassberger and Procaccia (1983), based on the work of Rényi (1955). The \(D_q\) of a multifractal measure is calculated as:

\[
D_q = \frac{\tau(q)}{q-1} = \lim_{\delta \to 0} \frac{\log[\sum_{i} \mu_i(\delta)\delta^{q}]}{\log \delta}, \quad q \neq 1 \quad (6a)
\]

and

\[
D_1 \approx \lim_{\delta \to 0} \frac{\sum_{i} \mu_i(\delta)\log[\mu_i(\delta)]}{\log \delta}, \quad q=1 \quad (6b)
\]

Equation (6a) shows that \(\tau(q)\) is also related to the generalized fractal dimension, \(D_q\). In fact, the concept of generalized dimension, \(D_q\), corresponds to the scaling exponent for the \(q^{th}\) moment of the measure. Using equation (6a), \(D_1\) becomes indeterminate. Therefore, for the particular case that \(q = 1\), equation (6b) was employed.

For a monofractal, \(D_q\) is a constant function of \(q\). However, for multifractal measures, the relationship between \(D_q\) and \(q\) is described by a S-shaped curve. In this case, the most frequently used generalized dimensions are \(D_0\) for \(q = 0\), \(D_1\) for \(q = 1\) and \(D_2\) for \(q = 2\), which are referred to as capacity, information (or Shannon entropy) and correlation dimension, respectively. The information dimension, \(D_1\), provides insight about the degree of heterogeneity in the distribution of the measure. The correlation dimension, \(D_2\), is associated to the uniformity of the measure among intervals and describes the average distribution density of the measure. In general, the generalized dimension, \(D_q\), is more useful for the comprehensive study of multifractals. Differences between \(D_q\) allow comparison of the complexity between measured soil water content data series. In homogeneous structures \(D_q\) are close, whereas in a monofractal they are equal.
3 Results and discussion

3.1 Patterns of vineyard soil water content under rain-fed and irrigation conditions

Temperatures for the two studied growing seasons were similar in average (Table 1); however, rainfall and evapotranspiration were higher in 2012. Harvest date was almost the same in both years. Nevertheless, the temporal evolution of rainfall and ETc differed from year to year (Fig. 2), being greater during 2012, especially at the beginning of the study period. This fact caused a different scheduling of irrigation between years.

Soil water content decreased over the growing season under rain-fed conditions in both years (Fig. 3). However, when irrigation was initiated, soil water content became more stable in the irrigated treatment (Fig. 3). The magnitude of the soil water loss was more evident in the layers of 20 and 40 cm depth, and less important in the 60 cm layer, which may indicate the depth of the active root zone as well as the intensity of root water uptake at each soil layer, as reported for other cultivars and crops (Intrigliolo and Castel, 2009; Mestas-Valero et al., 2011), and proved that FDR probes can be successfully used for irrigation scheduling (Goldhamer et al., 1999), calibrating them with established indicators such as midday stem water potential (Mirás-Avalos et al., 2014) and soil evaporation. Indeed, our results suggest that the water amount applied through irrigation was enough for fulfilling vineyard water requirements over the two growing seasons studied.

3.2 Multifractality of the soil water content time series

Soil water content time series obeyed power law scaling, as shown by the double log plots (Supplementary Fig. 4S.1). These plots allow to identify the range of moments needed to describe the scale variation of the studied parameter (Vidal Vázquez et al., 2010).

Figure 4 shows the partition functions for rain-fed and irrigation conditions at 20 cm depth in 2011. Visually, a slight departure from the straight line model was observed for moments $q < -1$ (Supplementary Fig. S.41). In general, higher deviations from linearity were found for the highest $q$ moments in the data series from the irrigation treatment, when compared to those from the rain-fed treatment, especially in 2012. Nevertheless, determination coefficients, $R^2$, were greater than 0.9 for statistical moments in the range from $q = -10$ to $q = 10$, in all the
studied data sets. Consequently, scalings are adequately defined. Similar results were found by Mestas-Valero et al. (2011) for soil water content under rain-fed grassland.

The $\tau(q)$ functions were different from a monofractal type of scaling for all series analyzed, especially under irrigation conditions (Supplementary Fig. 5S.2), similar to results obtained by Biswas et al. (2012b) for soil water storage. In fact, the heterogeneity of the soil water content data series from the irrigated treatment was greater than that of the rain-fed treatment (Supplementary Fig. 5S.2).

The value of $D_1$ is a good indicator of the heterogeneity degree in temporal distributions of a given variable. The closer the $D_1$ value to $D_0$, the more homogeneous is the distribution of the variable. In our case, rain-fed series were more homogeneous than the irrigated ones. In general, soil water content recorded at 60 cm depth presented the lower differences between $D_1$ and $D_0$ (Table 21), thus being more homogeneous both under rain-fed and irrigation conditions. Moreover, 2012 data series presented a higher heterogeneity than those from 2011 (Table 21) for both treatments, caused by the greater rainfall amount collected in 2012.

A monofractal would be characterized by $D_0 = D_1 = D_2$ (Evertsz and Mandelbrot, 1992). In all the studied data series $D_0 > D_1 > D_2$ (Table 21), indicating that soil water content had a tendency to behave as a multifractal. However, differences ($D_0 - D_1$) ranged from 0.051 to 0.222 and ($D_1 - D_2$) oscillated between 0.053 and 0.168, which suggests different degree in the homogeneity/heterogeneity of soil water content depending on the treatment imposed and the depth in the soil profile. In general, data series from the irrigation treatment showed greater differences between $D_0$, $D_1$ and $D_2$ than the series from the rain-fed treatment for both growing seasons. Moreover, the 60 cm depth layer presented smaller differences than the 20 and 40 cm layers (Table 21). The width of the $D_q$ spectra, determined by indicators such as $(D_0 - D_{10})$, showed different degrees of heterogeneity, with a trend to decrease in depth and under rain-fed conditions when compared with the irrigation treatment (Table 21). This is caused by the spiky nature of soil water content and indicates a multiple scaling nature at shallow depths. Moreover, the width of the $D_q$ spectra increased from 2011 to 2012 in both treatments, mainly in the 20 and 40 cm depths.

Generalized dimensions, or Rényi spectra, calculated for the range between $q = -10$ and $q = 10$ for soil water content data series at three depths under rain-fed and irrigation conditions are displayed on Fig. 64. All the data series studied showed Rényi spectra as asymmetric sigma-shaped curves with more curvature for the negative values of $q$ than for positive ones.
The left part of the curves is concave down and it changes to concave up on the right of the vertical axis. In the case of the soil water content series from the rain-fed treatment, the most curved spectra corresponded to the 40 cm depth data series, whereas for the irrigation treatment, the most curved one was the 20 cm depth data series (Fig. 64). When compared between treatments, Rényi spectra were more curved under irrigation conditions and the estimation errors were also greater under this treatment (Fig. 64). These results confirmed the higher heterogeneity (multifractality) of the data series from the irrigation treatment when compared to those from rain-fed.

Mestas-Valero et al. (2011) obtained monofractal distributions of soil water content time series under grassland when measured at depths greater than 40 cm, in contrast with our results. This disagreement is likely caused by the fact that grapevine root system reach greater depths than that of grass and vines are capable of uptaking water from deeper soil layers.

Determination coefficients, $R^2$, were highest for moments $q = 0$ and $q = 1$ and diminished for the other $|q|$ moments. In the case of $q = 10$, $R^2$ was greater than 0.97 and 0.95 in the rain-fed and irrigated data sets, respectively. For $q = -10$, $R^2$ values for rain-fed and irrigated data series were greater than 0.99 and 0.91, respectively (data not shown). Standard errors of $D_q$ values increased with increasing $|q|$ moments and they were much lower for right ($q > 0$) than for left ($q < 0$) branch of the Rényi spectra (Fig. 64).

Parameter $\alpha_0$ from the singularity spectra ranged from 1.056 to 1.146 in the rain-fed treatment and from 1.075 to 1.187 in the irrigated treatment (Table 32). The singularity spectrum allows for analyzing similarity or difference between the scaling properties of the measures as well as to assess the local scaling properties of soil water content measurements. The wider the spectrum is (i.e., the largest $\alpha_q - \alpha_q+$ value), the higher the heterogeneity in the scaling indices and vice versa (Vidal Vázquez et al., 2010). Moreover, the $f(\alpha)$ spectrum branch length gives insight about the abundance of the measure. Hence, small $f(\alpha)$ values at the end of a long branch correspond to rare events. Our results showed that the width of the singularity spectra increased in both treatments from 2011 to 2012 (Table 2).

Singularity spectra are characterized by a concave down shape (Fig. 75), showing an asymmetrical curve with wider but shorter right side. Rain-fed data series showed a shorter $f(\alpha)$ spectrum in both years, confirming their low degree of multifractality when compared to the irrigated data series (Fig. 75).
Differences ($\alpha_q - \alpha_0$ and $\alpha_0 - \alpha_{q^+}$) indicate the deviation of the spectrum from its maximum value ($q = 0$) towards the right side ($q < 0$) and the left side ($q > 0$), respectively (Vidal Vázquez et al., 2010). Usually, soil water content data series from the rain-fed treatment showed lower $\alpha_0 - \alpha_{q^+}$ values than those from the irrigated treatment (Table 32). Moreover, the highest values for this multifractal parameter were observed at 40 cm depth in both treatments and years (Table 32). This may indicate that higher soil water contents were more frequent under irrigation, being greater the differences between treatments at 40 cm depth in 2012. In contrast, the right branch ($\alpha_q - \alpha_0$) of the spectrum was usually wider for rain-fed conditions (Table 32). These results confirm the differential homogeneity/heterogeneity pattern between treatments evidenced by the generalized dimension, $D_q$, analysis (Table 21, Fig. 64).

4 Conclusions

Under the conditions of this study, continuous soil water content measurements at different depths reliably described the soil water balance in a vineyard over two irrigation periods. The logarithms of the partition function varied linearly with the logarithms of the time resolution for all the studied depths under both treatments considered in the range of moments $-10 < q < 10$, indicating that soil water content time series obeyed power laws.

The scaling properties of soil water content time series were reasonably fitted to multifractal models. These properties were different for the rain-fed and irrigation treatments, implying a higher heterogeneity for the data series from the irrigation treatment, which tended to increase in the second year of the study (2012). Therefore, multifractal analysis allowed us to discriminate among soil water content patterns in a vineyard for the 2011 and 2012 growing seasons as a function of irrigation use.

Author contribution

J. M. Mirás-Avalos and E. Trigo-Córdoba designed and carried out the field experiment. J. M. Mirás-Avalos, R. da Silva-Dias, I. Varela-Vila and A. García-Tomillo performed the analyses. J. M. Mirás-Avalos prepared the manuscript with contributions from all co-authors.
Acknowledgements

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References


Table 1. Summary of climate variables (temperature, rainfall and ET₀), irrigation water applied and harvest date for the studied period in 2011 and 2012 (from 14th June to 26 August):

<table>
<thead>
<tr>
<th>Year</th>
<th>Temperature (°C)</th>
<th>Rainfall (mm)</th>
<th>ET₀ (mm)</th>
<th>Irrigation (mm)</th>
<th>Harvest date</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
<td>Average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>12.44</td>
<td>28.86</td>
<td>20.15</td>
<td>230.78</td>
<td>14th September</td>
</tr>
<tr>
<td>2012</td>
<td>12.33</td>
<td>28.21</td>
<td>19.67</td>
<td>344.94</td>
<td>13th September</td>
</tr>
</tbody>
</table>
Table 21. Selected multifractal parameters: generalized dimensions, for the first-three positive moments, $D_0$, $D_1$, and $D_2$, with their respective errors of estimation, and two multifractality indices $\Delta(D_0 - D_2)$ and $\Delta(D_0 - D_{10})$.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Depth (cm)</th>
<th>$D_0$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$\Delta(D_0 - D_2)$</th>
<th>$\Delta(D_0 - D_{10})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rain-fed</td>
<td>20</td>
<td>0.999 ± 0.001</td>
<td>0.937 ± 0.008</td>
<td>0.884 ± 0.016</td>
<td>0.115</td>
<td>0.672</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.000 ± 0.000</td>
<td>0.881 ± 0.007</td>
<td>0.746 ± 0.014</td>
<td>0.254</td>
<td>0.752</td>
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<tr>
<td></td>
<td>60</td>
<td>1.000 ± 0.000</td>
<td>0.925 ± 0.007</td>
<td>0.868 ± 0.013</td>
<td>0.133</td>
<td>0.656</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>1.000 ± 0.000</td>
<td>0.916 ± 0.008</td>
<td>0.833 ± 0.019</td>
<td>0.167</td>
<td>0.589</td>
</tr>
<tr>
<td>Irrigated</td>
<td>20</td>
<td>0.999 ± 0.001</td>
<td>0.868 ± 0.013</td>
<td>0.778 ± 0.026</td>
<td>0.221</td>
<td>0.757</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1.000 ± 0.000</td>
<td>0.852 ± 0.019</td>
<td>0.773 ± 0.026</td>
<td>0.227</td>
<td>0.698</td>
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<tr>
<td></td>
<td>60</td>
<td>1.000 ± 0.000</td>
<td>0.852 ± 0.022</td>
<td>0.758 ± 0.034</td>
<td>0.242</td>
<td>0.664</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>1.000 ± 0.000</td>
<td>0.861 ± 0.023</td>
<td>0.773 ± 0.037</td>
<td>0.227</td>
<td>0.695</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rain-fed</td>
<td>20</td>
<td>0.999 ± 0.001</td>
<td>0.861 ± 0.014</td>
<td>0.771 ± 0.025</td>
<td>0.228</td>
<td>0.856</td>
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<tr>
<td></td>
<td>40</td>
<td>1.000 ± 0.000</td>
<td>0.888 ± 0.008</td>
<td>0.739 ± 0.017</td>
<td>0.261</td>
<td>0.801</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1.000 ± 0.000</td>
<td>0.949 ± 0.004</td>
<td>0.907 ± 0.005</td>
<td>0.093</td>
<td>0.548</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>1.000 ± 0.000</td>
<td>0.898 ± 0.006</td>
<td>0.768 ± 0.016</td>
<td>0.232</td>
<td>0.682</td>
</tr>
<tr>
<td>Irrigated</td>
<td>20</td>
<td>0.984 ± 0.006</td>
<td>0.831 ± 0.010</td>
<td>0.731 ± 0.019</td>
<td>0.253</td>
<td>1.024</td>
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<tr>
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<td>40</td>
<td>0.979 ± 0.006</td>
<td>0.757 ± 0.014</td>
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<td>0.390</td>
<td>1.210</td>
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<tr>
<td></td>
<td>60</td>
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<td>0.907 ± 0.007</td>
<td>0.805 ± 0.015</td>
<td>0.195</td>
<td>0.622</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>0.993 ± 0.003</td>
<td>0.822 ± 0.016</td>
<td>0.707 ± 0.030</td>
<td>0.286</td>
<td>1.085</td>
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</table>
Table 32. Selected multifractal parameters derived from the $f(\alpha)$ singularity spectra: most positive ($q_+$) and most negative ($q_-$) limits the range of multifractal scaling, Hölder exponent of order 0 ($\alpha_0$), most positive ($\alpha_{q^+}$) and most negative ($\alpha_{q^-}$) exponents, widths of the left ($\alpha_0 - \alpha_{q^+}$) and the right ($\alpha_{q^-} - \alpha_0$) sides of the spectra.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Depth (cm)</th>
<th>$q_-$</th>
<th>$q_+$</th>
<th>$\alpha_0$</th>
<th>$\alpha_{q^+}$</th>
<th>$\alpha_{q^-}$</th>
<th>$\alpha_0 - \alpha_{q^+}$</th>
<th>$\alpha_{q^-} - \alpha_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rain-fed</td>
<td>20</td>
<td>-1.5</td>
<td>3.5</td>
<td>1.066</td>
<td>0.768</td>
<td>1.339</td>
<td>0.299</td>
<td>0.273</td>
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<tr>
<td></td>
<td>40</td>
<td>-3.5</td>
<td>2</td>
<td>1.093</td>
<td>0.632</td>
<td>1.328</td>
<td>0.460</td>
<td>0.235</td>
</tr>
<tr>
<td></td>
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<td>-3.5</td>
<td>2</td>
<td>1.087</td>
<td>0.718</td>
<td>1.403</td>
<td>0.369</td>
<td>0.315</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>-4</td>
<td>2</td>
<td>1.074</td>
<td>0.762</td>
<td>1.297</td>
<td>0.312</td>
<td>0.222</td>
</tr>
<tr>
<td>Irrigated</td>
<td>20</td>
<td>-2.5</td>
<td>2</td>
<td>1.136</td>
<td>0.714</td>
<td>1.450</td>
<td>0.422</td>
<td>0.314</td>
</tr>
<tr>
<td></td>
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<td>-4</td>
<td>3</td>
<td>1.160</td>
<td>0.664</td>
<td>1.383</td>
<td>0.496</td>
<td>0.222</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-5</td>
<td>2</td>
<td>1.132</td>
<td>0.700</td>
<td>1.333</td>
<td>0.435</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>-4.5</td>
<td>2</td>
<td>1.142</td>
<td>0.709</td>
<td>1.375</td>
<td>0.433</td>
<td>0.233</td>
</tr>
<tr>
<td>Rain-fed</td>
<td>20</td>
<td>-2.5</td>
<td>3</td>
<td>1.146</td>
<td>0.659</td>
<td>1.526</td>
<td>0.487</td>
<td>0.380</td>
</tr>
<tr>
<td></td>
<td>40</td>
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<td>2</td>
<td>1.082</td>
<td>0.603</td>
<td>1.301</td>
<td>0.479</td>
<td>0.219</td>
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<tr>
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<td>-2</td>
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<td>0.746</td>
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<td>0.309</td>
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<tr>
<td></td>
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<td>0.651</td>
<td>1.265</td>
<td>0.426</td>
<td>0.188</td>
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<tr>
<td>Irrigated</td>
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<td>-0.5</td>
<td>2.5</td>
<td>1.164</td>
<td>0.602</td>
<td>1.361</td>
<td>0.562</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>-1</td>
<td>1.5</td>
<td>1.187</td>
<td>0.575</td>
<td>1.491</td>
<td>0.611</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>-4</td>
<td>2</td>
<td>1.075</td>
<td>0.716</td>
<td>1.223</td>
<td>0.360</td>
<td>0.148</td>
</tr>
<tr>
<td></td>
<td>20-60</td>
<td>-1</td>
<td>2</td>
<td>1.172</td>
<td>0.624</td>
<td>1.489</td>
<td>0.548</td>
<td>0.317</td>
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</tbody>
</table>
Figure 1. Location of the studied vineyard and experimental layout.
Figure 2. Crop evapotranspiration ($ET_c$), rainfall and irrigation water applied over the two growing seasons studied, 2011 and 2012. Day of the year 166 is 14th June.
Figure 3. Soil water content at three depths (20, 40 and 60 cm) for rain-fed and irrigation treatments over the 2011 and 2012 growing seasons. DOY stands for Day of the Year (165 = 13th June).
Figure 4. Selected plots of the natural logarithms of the partition function, $\chi(q, \delta)$, versus the time resolution, $\delta$. a) rain-fed treatment at 20 cm depth in 2011; b) irrigated treatment at 20 cm depth in 2011.
Figure 5. Mass exponents, $\tau(q)$, of soil water content averaged from 20 to 60 cm depth for rain-fed and irrigation treatments: a) 2011, b) 2012.
Figure 64. Generalized dimension, $D_q$, spectra (-10 < $q$ < 10) of soil water content for rain-fed and irrigation treatments at the studied depths in 2011 and 2012. Bars indicate estimation errors.
Figure 75. Singularity spectra for soil water content averaged from 20 to 60 cm depth for rain-fed and irrigation treatments in 2011 and 2012: a) 2011, b) 2012.
**Multifractal behaviour of the soil water content of a vineyard in NW Spain during two growing seasons. Mirás-Avalos et al. (Supplementary Materials)**

**Supplementary information**

Table S1. Summary of climate variables (temperature, rainfall, $ET_0$ and $ET_c$), irrigation water applied and harvest date for the studied and irrigation periods in 2011 and 2012.

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
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<tr>
<td><strong>Measurement period</strong></td>
<td>14 June to 26 August</td>
<td>14 June to 26 August</td>
</tr>
<tr>
<td>Temperature ($^\circ$C)</td>
<td></td>
<td></td>
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<tr>
<td>Maximum</td>
<td>28.86</td>
<td>28.21</td>
</tr>
<tr>
<td>Minimum</td>
<td>12.44</td>
<td>12.33</td>
</tr>
<tr>
<td>Mean</td>
<td>20.15</td>
<td>19.67</td>
</tr>
<tr>
<td>Rainfall (mm)</td>
<td>25.60</td>
<td>65.60</td>
</tr>
<tr>
<td>$ET_0$</td>
<td>230.78</td>
<td>344.91</td>
</tr>
<tr>
<td>$ET_c$</td>
<td>184.63</td>
<td>275.93</td>
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<tr>
<td><strong>Irrigation period</strong></td>
<td>9 July to 16 August</td>
<td>20 July to 22 August</td>
</tr>
<tr>
<td>Temperature ($^\circ$C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>29.14</td>
<td>30.37</td>
</tr>
<tr>
<td>Minimum</td>
<td>12.61</td>
<td>12.77</td>
</tr>
<tr>
<td>Mean</td>
<td>20.36</td>
<td>20.82</td>
</tr>
<tr>
<td>Rainfall (mm)</td>
<td>10.20</td>
<td>29.20</td>
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<td>$ET_0$</td>
<td>117.86</td>
<td>162.52</td>
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<tr>
<td>$ET_c$</td>
<td>94.28</td>
<td>130.02</td>
</tr>
<tr>
<td>Irrigation (mm)</td>
<td>39.67</td>
<td>50.00</td>
</tr>
<tr>
<td>Harvest date</td>
<td>14$^{th}$ September</td>
<td>13$^{th}$ September</td>
</tr>
</tbody>
</table>
**Figure S1.** Selected plots of the natural logarithms of the partition function, $\chi(q,\delta)$, versus the time resolution, $\delta$: a) rain-fed treatment at 20 cm depth in 2011; b) irrigated treatment at 20 cm depth in 2011.
Figure S2. Mass exponents, $\tau(q)$, of soil water content averaged from 20 to 60 cm depth for rain-fed and irrigation treatments in 2011 and 2012.