

**Interactive comments on “Spatial and radiometric characterization of multi-spectrum satellite images through multifractal analysis” by Carmelo Alonso et al.**

**Answer to Anonymous Referee #2**

Although the authors' similarity report found some similarity with other papers by the authors, my main concern was a lack of similarity with one of them! Indeed, the authors completely failed to cite the first and still the most comprehensive multifractal analysis of satellite derived vegetation indices. Without this, their own paper is without adequate context, their results are simply isolated numbers – and as we argue – the numbers that are kept are stochastic variables and hence will lack reproduce ability. This failure is remarkable because the second author of the paper was a key author in the earlier more thorough and quantitative one the same subject.

Apologize for this mistake. We are totally agreed that we should include the paper you mention and the second author specially apologize for this mistake. The new version of this manuscript has included now the reference you mention:

Lovejoy , S., A. Tarquis, H. Gaonac'h, and D. Schertzer (2008), Single and multiscale remote sensing techniques, multifractals and MODIS derived vegetation and soil moisture, *Vadose Zone J.*, 7, 533-546 doi: doi 10.2136/vzj2007.0173.

We had very few time to deliver this manuscript to be included in this special issue. We really appreciate to the editors the opportunity they gave us.

As we normally do in scientific context, each time that you refer to this paper, we will mention henceforth as Lovejoy paper, as Shaun Lovejoy is the first author of this paper.

In this paper, the authors use the multifractal *dimension* formalism of Halsey el 1986 that was developed for characterizing the deterministic phase spaces of strange attractors. In [*Lovejoy et al.*, 2008] co-authored by the second author in the present paper A. Tarquis (henceforth the “Lovejoy paper”), it was explained in considerable detail why the dimension formalism is ill-suited for stochastic multifractals. Here, the images are assumed to be densities of multifractal measures, each realizations of a stochastic process. Co-author A. Tarquis can surely explain why the co-dimension formalism is more appropriate for the present application. She can also explain why the assumption of the existence of Holder exponents does not generally hold for stochastic multifractals and how the co-dimension formalism avoids this unnecessary (and doubt fully valid) assumption.

The authors are working already on it to dedicate another manuscript to make a comparison of both methodologies on these images. This is an important issue that deserve a manuscript just with this aim, as it has been done in Morato et al. and Renosh et al. papers:

Morató, M.C., M.T. Castellanos, N.R. Bird, A.M. Tarquis. Multifractal analysis in soil properties: Spatial signal versus mass distribution. *Geoderma*, 287, 54-65, 2017. <http://dx.doi.org/10.1016/j.geoderma.2016.08.004>.

Renosh, P. R., Schmitt, F. G., and Loisel, H. 2015. Scaling analysis of ocean surface turbulent heterogeneities from satellite remote sensing: use of 2D structure functions. *PLoS ONE*, 10, e0126975. doi:10.1371/journal.pone.0126975.

At the very least the paper must acknowledge the existence of the codimension formalism and refer to the Lovejoy paper. The authors should also give the formulae:

where  $d$  is the dimension of space (here  $d = 2$ ) and  $c(\gamma)$  is the codimension of the singularity of the density of the multifractal measure  $\gamma$  ( $\gamma$  is related to the singularity of the measure  $\alpha$  by the formula above) and  $K(q)$  is the moment scaling function of the density of the multifractal measure (i.e. it directly characterizes the scaling of the moments of the image rather than the integral of the image). These formulae are necessary in order to compare results obtained in the two formalisms (i.e. with the rest of the literature).

Now we have included in Material and Methods the followed in the subsection of Multifractal analysis:

A monofractal object can be measured by counting the number  $N$  of  $\delta$  size boxes needed to cover the object. The measure depends on the box size as

$$N(\delta) \propto \delta^{-D_0} \quad (3)$$

where

$$D_0 = \lim_{\delta \rightarrow 0} \frac{\log N(\delta)}{\log \frac{1}{\delta}} \quad (4)$$

is the fractal dimension.  $D_0$  is calculated from slope of a log-log plot. However, many examples are found where a single scaling law cannot be applied and it is necessary to do a multiscaling analysis.

There are several methods for implementing multifractal analysis. The Universal Multifractal (UM) model assumes that multifractals are generated from a random variable with an exponentiated extreme Levy distribution (Lavallée et al., 1991; Tessier et al., 1993). In UM

analysis, the scaling exponent  $K(q)$  is highly relevant. This function for the moments  $q$  of a cascade conserved process is obtained according to Schertzer and Lovejoy (1987) as follows:

$$\text{---} \tag{5}$$

where  $C_1$  is the mean intermittency codimension and  $\alpha$  is the Levy index. These are known as the UM parameters.

Other method is the moment method developed by Halsey et al. (1986) and applied to this case study. This method uses mainly three functions:  $\tau(q)$ , known as the mass exponent function,  $\alpha$ , the coarse Hölder exponent, and  $f(\alpha)$ , multifractal spectrum. A measure (or field), defined in two-dimensional image embedding space ( $n \times n$  pixels) and with values based on grey tones (for 8 bits goes from 0 to 255), cannot be consider as a geometrical set and therefore cannot be characterized by a single fractal dimension.

To characterize the scaling property of a variable measured on the spatial domain of the studied, it divides the image into a number of self-similar boxes. Applying disjoint covering by boxes in an “up-scaling” partitioning process we obtain the partition function  $\chi(q, \delta)$  (Feder, 1989) defined as:

$$\chi(q, \delta) = \sum_{i=1}^{N(\delta)} \mu_i^q(\delta) = \sum_{i=1}^{N(\delta)} m_i^q \tag{6}$$

where  $m$  is the mass of the measure,  $q$  is the mass exponent,  $\delta$  is the length size of the box and  $N(\delta)$  is the number of boxes in which  $m_i > 0$ . Based on this, the mass exponent function  $\tau(q)$  shows how the moments of the measure scales with the box size:

$$\tau(q) = \lim_{\delta \rightarrow 0} \frac{\log \langle \chi(q, \delta) \rangle}{\log(\delta)} = \lim_{\delta \rightarrow 0} \frac{\log \langle \sum_{i=1}^{N(\delta)} m_i^q \rangle}{\log(\delta)} \tag{7}$$

where  $\langle \rangle$  represents statistical moment of the measure  $\mu_i(\delta)$  defined on a group of non overlapping boxes of the same size partitioning the area studied.

The singularity index,  $\alpha$ , can be determined by the Legendre transformation of the  $\tau(q)$  curve (Halsey, 1986) as:

$$\alpha(q) = \frac{d\tau(q)}{dq} \tag{8}$$

The number of cells of size  $\delta$  with the same  $\alpha$ ,  $N_\alpha(\delta)$ , is related to the cell size as  $N_\alpha(\delta) \propto \delta^{-f(\alpha)}$ , where  $f(\alpha)$  is a scaling exponent of the cells with common  $\alpha$ . Parameter  $f(\alpha)$  can be calculated as:

$$f(\alpha) = q\alpha(q) - \tau(q) \tag{9}$$

Multifractal spectrum (MFS) shown as plot of  $\alpha$  vs.  $f(\alpha)$ , quantitatively characterizes variability of the measure studied with asymmetry to the right and left indicating domination of small and large values respectively (Evertsz and Mandelbrot, 1992). There are three characteristic values obtained from MFS, the singularity  $\alpha(q)$  values for  $q = \{0, 1, 2\}$ . The first value ( ) corresponds to the maximum of MFS and it is related to the box-counting dimension of the measure support; the second value is related to information or entropy dimension ( ) and the third with the correlation dimension. The entropy dimension quantifies the degree of disorder present in a distribution. According to Andraud et al. (1994) and Gouyet (1996) a value close to 2.0 characterizes a system uniformly distributed throughout all scales, whereas a value close to 0 reflects a subset of the scale in which the irregularities are concentrated. These three values will be shown from each calculation of MFS.

The width of the MF spectrum ( ) indicates overall variability (Tarquis et al., 2001; 2014) and we have split it in two sections. Section I correspond to values  $\alpha(q) < \alpha(0)$  or  $q > 0$  and section II to values with  $\alpha(q) > \alpha(0)$  or  $q < 0$ . In section I the amplitude, or semi-width, was calculated with differences  $\Delta^+ = \alpha(0) - \alpha(+5)$ , and in section II with  $\Delta^- = \alpha(-5) - \alpha(0)$ .

To study the asymmetry of the multifractal spectrum we have choose the asymmetry index (AI) estimated as (Xie et al., 2010):

$$\text{AI} = \frac{\Delta^+}{\Delta^-} \tag{10}$$

In our case, is the singularity for  $q=0$  or , is and is . Therefore, we can rewrite as:

(11)

Expressing as equation (11), we can see that it is a normalized index based on the amplitudes and .

There are several works relating the UM model and the multifractal formalism based on  $\tau(q)$  (Gagnon et al., 2003; Aguado et al., 2014; Morató et al., 2017 among others) through the equations:

(12)

(13)

where  $E$  is the Euclidean dimension where the measure is embedded, in this case will be  $E=2$ , and  $c(\gamma)$  is the codimension of the singularity of the density of the multifractal measure  $\gamma$ .

One of the advantages of the codimension formalism is immediately obvious from the formulae:  $c(\gamma)$ ,  $K(q)$  are independent of the dimension of the embedding space  $d$  whereas  $f(\alpha)$ , are different where ever one looks at subspaces of the process (i.e. the same process but observed at different  $d$ ). An related advantage of the codimension formalism is that when one performs the moment analysis (e.g. their figs 3, 6) that the moments will not dominated by the trivial, deterministic scaling factor but will directly show the key (and usually much smaller) part see the expression above; such an analysis is called “trace moment analysis”). As it is, the quality of the scaling of the statistics is practically impossible to judge from the authors’ figures. In addition - also as explained in the Lovejoy paper – the moments  $q < 0$  will in general diverge so that special care is needed to avoid spurious estimates.

As carefully explained in the Lovejoy paper, the multifractal spectrum  $f(\alpha)$  – or better,  $c(\gamma)$  - is a function; empirically it corresponds to estimating an infinite number of parameters. Since the framework is of stochastic processes, and in general stochastic multifractals have unbounded spectra (i.e.  $c(\gamma)$  is generally unbounded), the authors differences  $\Delta_{\pm}$  are simply random variables, they will provide very poor characterizations of the process. Why don’t the authors characterize the multifractality as explained in the Lovejoy paper (using  $C_1$ , and the multifractal index  $\alpha$  - not the same as the authors’  $\alpha$ )? An added bonus would be that they could quantitatively compare their results with others in the literature (including those in the Lovejoy paper!), rather than simply obtaining an isolated result with no context, no point of comparison. There are other ways of quantitatively characterizing the multifractality, but the singularity range used here is a particularly poor choice.

We understand that you prefer the UM model than the multifractal spectrum and perhaps you consider the later a poor choice. However, the results are similar than the one found in Lovejoy paper. A quantitative comparison of both methodologies it will be the aim of our next manuscript where we can study deeper why discrepancies or agreements as it has been done in Morato et al. (2017) paper on a transect data of soil properties and Renosh et al. (2015) work applied on 2D remote sensing images. We appreciate these comments that will help us to improve the discussion in this next manuscript. Also it will be interesting to compare with the Structure Function and Detrended Fluctuation Analysis, other methods that we haven't mentioned here.

As mentioned in Morato et al. (2017) work introduction, the methodology we have applied here is the most common used in Soil Science for several reasons, and that is why we began to use it in this manuscript. Just looking into the NPG journal we can found several articles with this methodology used.

We agree that we shouldn't stop here and applied other type of methodologies that could be more interesting. We have added the follow at the end of Conclusions:

“Further research will be conducted to establish a qualitative and quantitative comparison of these conclusions among several multifractal methodologies applied on these images.”

## References

Morató, M.C. , M.T. Castellanos, N.R. Bird, A.M. Tarquis. Multifractal analysis in soil properties: Spatial signal versus mass distribution. *Geoderma*, 287, 54-65, 2017. <http://dx.doi.org/10.1016/j.geoderma.2016.08.004>.

Renosh, P. R., Schmitt, F. G., and Loisel, H. 2015. Scaling analysis of ocean surface turbulent heterogeneities from satellite remote sensing: use of 2D structure functions. *PLoS ONE*, 10, e0126975. doi:10.1371/journal.pone.0126975.

**Another problem with the authors' characterization technique is that it ignores the issue of multifractal phase transitions that is extensively dealt with in the Lovejoy paper. The authors should check that their moments (up to the rather high value of  $q = 5$ ) are not spurious.**

We agree that higher is  $q$  value the errors could increase considerable. There are many works in Soil Science using this multifractal methodology that are applied from  $q=-10$  to  $q=+10$ . For this reason we only included a range of 5 ( $q=-5$  till  $q=+5$ ). The errors of the  $\alpha(q)$  values are included in Table 1, 2 and 3.

Some of the other conclusions of the Lovejoy paper could also be recalled and the authors' new results could be then be quantitatively compared.

Now in Results and Discussion these conclusions of Lovejoy paper are recalled. In the next manuscript we are going to compare both conclusions, in a quantitative

and qualitative way, based on the same images following the line of Morato et al. (2016) and Renosh et al. (2015) works.

## References

Morató, M.C., M.T. Castellanos, N.R. Bird, A.M. Tarquis. Multifractal analysis in soil properties: Spatial signal versus mass distribution. *Geoderma*, 287, 54-65, 2017. <http://dx.doi.org/10.1016/j.geoderma.2016.08.004>.

Renosh, P. R., Schmitt, F. G., and Loisel, H. 2015. Scaling analysis of ocean surface turbulent heterogeneities from satellite remote sensing: use of 2D structure functions. *PLoS ONE*, 10, e0126975. doi:10.1371/journal.pone.0126975.

**Conclusion:** This paper should not be published without proper citations and comparisons with the Lovejoy paper.

In this conclusion we are partially agree. Of course, as mentioned earlier the citations are already included about Lovejoy paper and the ones related to other methodology to estimate the multifractality. Also, we have included in the new version a qualitative comparison of the results on the common bands and NDVI showed in Lovejoy paper.

The quantitative comparison of both methodologies is a work in progress already following the line that was developed in the paper Morató et al. (2017) but extending it for 2D.

## Reference

M.C. Morató, M.T. Castellanos, N.R. Bird, A.M. Tarquis. Multifractal analysis in soil properties: Spatial signal versus mass distribution. *Geoderma*, 287, 54-65, 2017. <http://dx.doi.org/10.1016/j.geoderma.2016.08.004>.

## Detailed Comments:

Section 2.2, line 2: The authors state:

“A multifractal analysis is basically the measurement of a statistic distribution and therefore gives useful information even if the underlying structure does not show a full self similar behaviour (Plotnick et al., 1996).”

This is incomprehensible since isotropic multifractals assumed to be self-similar (i.e. scaling and isotropic), and the authors do not consider anisotropy in this paper. It is more correct to say that: “A multifractal analysis is an analysis of how the statistical properties of a scaling field (or series) varies with scale. It therefore does *not* give useful information when the underlying structure is not scaling.”

Thank you so much for your comment; we have delete that paragraph to don't create confusion.

## References:

**Lovejoy , S., A. Tarquis, H. Gaonac'h, and D. Schertzer (2008), Single and multiscale remote sensing techniques, multifractals and MODIS derived vegetation and soil moisture, *Vadose Zone J.*, 7, 533-546 doi: doi 10.2136/vzj2007.0173.**

**Lovejoy paper already included. Thanks so much to help us to avoid this mistake.**