A signal is stationary over a given observation scale if its spectrum undergoes no evolution in that scale. This assumption leads Bayram and Baraniuk (2000) to use Multitaper Spectrograms (MS) for studying the time-dependent features of signals as

\[
\omega_x(t,f) = \frac{1}{K} \sum_{k=0}^{K} \int x(t) h_k(-t) e^{-j2\pi ft} dt
\]

(1)

where \(h_k(t)\) stands for the first \(K\) Hermite functions, which are used as the short-length windows. Bayram and Baraniuk (2000) used the Hermite functions \(h_k^H(t)\) as the sliding windows since they give the best time-frequency localization and orthonormality in the time-frequency domain. Hermite functions can be obtained recursively, as follows

\[
h_k^H(t) = \frac{\pi^{-1} (2^k k!)^{1/2}}{e^{-t^2/2}} H_k(t)
\]

(2)

where \(H_k(t), t \in \mathbb{N}\) represents Hermite polynomials, defined by

\[
H_k(t) = 2tH_{k-1}(t) - 2(k-2)H_{k-2}(t)
\]

(3)
in which \(H_0(t) = 1\) and \(H_1(t) = 2t\). These family of windows are mutually orthonormal with elliptic symmetry and maximum concentration in the time-frequency domain. To define the global spectrum of signal, we should take the average of MS as (Xiao et al., 2007)

\[
\langle \omega_x(t,f) \rangle_N = \frac{1}{N} \sum_{t=0}^{N} \omega_x(t,f)
\]

(4)

For a stationary signal \(\omega_x(t,f)/\omega_x^{av}(t,f)\) remains almost unchanged at the whole recording window, but in practice fluctuations in this ratio is inevitable. These fluctuations can be defined by a dissimilarity function as

\[
c^t_i = D(\omega_x(t,f), \omega_x^{av}(t,f)), t = 0, ..., N
\]

(5)

The significance of fluctuations can also be assessed by using surrogates (Borgnont et al., 2010). A surrogate is artificially produced in such a way that mimics statistical properties of real data. Isospectral surrogates have identical power spectra as the real signal but with randomized phases (Theiler et al., 1992). Once a collection of \(J\) synthesized isospectral surrogates, \(\{s_j(t), j = 1, ..., J\}\), are generated, the dissimilarity between local, \(\omega_{s_j}(t,f)\), and global spectra, \(\omega_x^{av}(t,f)\), for surrogates can be evaluated by (Borgnont et al., 2010)

\[
\{c^j_t = D(\omega_{s_j}(t,f), \omega_x^{av}(t,f)), t = 0, ..., N, j = 1, ..., J\}
\]

(6)

Borgnont et al., (2010) merged the Kullback-Leibler distance,

\[
D_{KL}(A,B) = \int_{\Omega} (A(f) - B(f)) \log(A(f)/B(f)) df
\]

(7)

and log-spectral distance, \(D_{LSD}(A,B)\),

\[
D_{LSD}(A,B) = \int_{\Omega} |\log(A(f)/B(f))| df
\]

(8)
in the following combined form

\[
D(A,B) = D_{KL}(A,B) \left(1 + D_{LSD}(\hat{A},\hat{B})\right)
\]

(9)

In these equations \(A\) and \(B\) are two positive distributions and \(\hat{A}\) and \(\hat{B}\) indicate their normalized versions to the unity over the domain. The dissimilarity function \(D(A,B)\) enables us to differentiate an amplitude-modulated or frequency-modulated non-stationary signal from a stationary one. Statistical variance \(\Theta_i = var(c^i_n)_{n=1,...,N}\) gives the variance of \(c^i_n\)’s. Similarly, for each one of \(J\) synthesized surrogates we can define a separate variance as

\[
\{\Theta_0(j) = var(c^j_n)_{n=1,...,N'}, j = 1, ..., J\}
\]

(10)

These \(\Theta_0\)’s can be assumed as a set of realizations of Gamma probability distribution with the following description...
\[ P(x; a, b) = \frac{1}{b^a a^{a-1}} x^{a-1} \exp(-\frac{x}{b}) \]  

(11)

As a null hypothesis original signals is supposed to be stationary but if it violates the predefined threshold \( y \), null hypothesis is rejected and non-stationarity is assumed, that is

\[ J(x) = \begin{cases} 
1 & \text{if } \theta_1 > y: \text{ non-stationarity} \\
0 & \text{if } \theta_1 < y: \text{ stationarity} 
\end{cases} \]  

(12)

The threshold value for \( y \) is considered as a confidence level of 95\% for probability distribution under the maximum likelihood sense. By comparing \( \theta_1 \) and the estimates of \( \theta_0 \), one can define the degree of stationarity. Quantitatively, these difference can be evaluated by index of non-stationarity (INS) (Xiao et al., 2007):

\[ \text{INS} = \sqrt{\frac{1}{J} \sum_{n=1}^{J} \theta_0(j)} \]  

(13)

Further, note the result of stationarity test depends on the window length of spectrogram, \( T_n \). This dependence can be analyzed by the scale of non-stationarity (SNS). It informs us that in which one/ones of considered values for \( T_n \) the given threshold in Eq. (10) has been exceeded (Xiao et al., 2007):

\[ \text{SNS} = \frac{1}{J} \arg \max_{T_n} \{ \text{INS}(T_n) \} \]  

(14)