



1 **Can the Nucleation Phase be Generated on a Sub-fault**  
2 **Linked to the Main Fault of an Earthquake?**

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8

9 **Abstract.** We study the effects of seismic coupling, friction, viscous, and inertia on  
10 earthquake nucleation based on a two-body spring-slider model in the presence of  
11 thermal-pressurized slip-dependent friction and viscosity. The stiffness ratio of the  
12 system to represent seismic coupling is the ratio of coil spring  $K$  between two sliders  
13 and the leaf spring  $L$  between a slider and the background plate and denoted by  $s=K/L$ .  
14 The  $s$  is not a significant factor in generating the nucleation phase. The masses of the  
15 two sliders are  $m_1$  and  $m_2$ , respectively. The frictional and viscous effects are  
16 specified by the static friction force,  $f_0$ , the characteristic displacement,  $U_c$ , and  
17 viscosity coefficient,  $\eta$ , respectively. Numerical simulations show that friction and  
18 viscosity can both lengthen the natural period of the system and viscosity increases  
19 the duration time of motion of the slider. Higher viscosity causes lower particle  
20 velocities than lower viscosity. The ratios  $\gamma=\eta_2/\eta_1$ ,  $\phi=f_{o2}/f_{o1}$ ,  $\psi=U_{c2}/U_{c1}$ , and  
21  $\mu=m_2/m_1$  are four important factors in influencing the generation of a nucleation  
22 phase. When  $s>0.17$ ,  $\gamma>1$ ,  $1.15>\phi>1$ ,  $\psi<1$ , and  $\mu<30$ , simulation results exhibit the  
23 generation of nucleation phase on slider 1 and the formation of  $P$  wave on slider 2.  
24 The results are consistent with the observations and suggest the possibility of



25 generation of nucleation phase on a sub-fault.

26

27 **Keywords:** nucleation phase, two-body spring-slider model, stiffness ratio, thermal-

28 pressurized slip-dependent friction, viscosity

29



## 30 1 Introduction

31 The presence of nucleation phase before the  $P$  waves (see Fig. 1) was suggested by  
32 early theoretical studies (e.g., Andrews, 1976; Brune, 1979; Dieterich, 1986, 1992;  
33 Das and Scholz, 1981) and laboratory experiments (Dieterich, 1979; Ohnaka et al.,  
34 1987). Some studies (Scholz et al., 1972; Dieterich, 1981; Ohnaka and Yamashita,  
35 1989; Ohnaka, 1992; Ohnaka and Kuwahara, 1990; Kato et al., 1994; Roy and  
36 Marone, 1996; Lu et al., 2010; Latour et al., 2013; Kaneko et al., 2016) also indicated  
37 that the nucleation process behaves like a transition from quasi-static slip (without the  
38 inertial effect) to (unstable) dynamic motion (with the inertial effect) when the slip  
39 speeds become high enough to make the inertial effect dominate frictional resistance  
40 under some conditions. The study of this phase is a basic problem of earthquake  
41 physics and also important for early warning, prediction, and hazard assessment of  
42 earthquakes.

43 Umeda (1990) first recognized the nucleation phase in velocity seismograms.  
44 Since then, numerous seismologists also observed the nucleation phases (e.g., Iio,  
45 1992, 1995; Ellsworth and Beroza, 1995; Beroza and Ellsworth, 1996; Mori and  
46 Kanamori, 1996; Ruiz et al., 2017). There is a debate concerning the correlation  
47 between the duration time,  $T_D$ , of nucleation phase and the magnitude,  $M$ , of the  
48 earthquake occurring immediately after the nucleation phase. Ellsworth and Beroza  
49 (1995) and Beroza and Ellsworth (1996) assumed a positive correlation of  $T_D$  to  $M$ .  
50 Whereas, Mori and Kanamori (1996) observed independence of the  $P$  waves on the  
51 shape of nucleation phase in a large magnitude range. Ellsworth and Beroza (1998)  
52 confirmed the observation by Mori and Kanamori (1996).

53 Friction and viscosity are two major factors in controlling the complicated  
54 earthquake rupture processes including nucleation (see Wang, 2016; and cited



55 references therein). Analytic solutions and numerical simulations for exploring the  
56 nucleation phase have made based on the infinite dislocation models, crack models,  
57 and spring-slider models by using different friction laws (see Beeler, 2004; Tal et al.,  
58 2018; Wang, 2016, 2017a; and cited references therein). Iio (1992, 1995) stressed that  
59 the nucleation phase cannot be interpreted by any theoretical source model with a  
60 constant kinematic friction and a constant rupture velocity. Mori and Kanamori (1996)  
61 claimed that any model having a similar initial rupture can describe the nucleation  
62 phases of earthquakes of all sizes, and thus it is difficult to estimate the magnitude of  
63 an earthquake just from its nucleation phase. They also stressed that curvature seen in  
64 the nucleation phases is caused by anelastic attenuation.

65 Some theoretical studies based on the Burridge-Knopoff spring-slider model  
66 (Burridge and Knopoff, 1967), from which the two-body model used in this study is  
67 simplified, are briefly described here. Brantut et al. (2011) concluded that  
68 metamorphic dehydration influences the nucleation of unstable slip and could be an  
69 origin for slow-slip events in subduction zones. Ueda et al. (2014, 2015) and  
70 Kawamura et al. (2018) pointed out that the nucleation process includes the quasi-  
71 static initial phase, the unstable acceleration phase, and the high-speed rupture phase  
72 (i.e., a mainshock) and recognized two kinds of nucleation lengths, i.e.,  $L_{sc}$  and  $L_c$   
73 which are affected by model parameters, yet not by the earthquake size. The  $L_{sc}$   
74 related to the initial phase exists only for a weak frictional instability regime; while  
75 the  $L_c$  associated with the acceleration phase exist for both weak and strong instability  
76 regimes. They also found that in the initial phase up to  $L_{sc}$ , the sliding velocity is of  
77 order the plate speed, while at a certain stage of the acceleration phase it becomes  
78 higher and thus can be observed.

79 Although the frictional effect on earthquake nucleation has been long and widely



80 studied as mentioned above, the studies of viscous effect on earthquake ruptures are  
81 rare. The viscous effect mentioned in Rice et al. (2001) was actually an implicit factor  
82 which is included within the direct effect of rate- and state-dependent friction law.  
83 Wang (2017a) took viscosity into account for studying the nucleation phase by  
84 assuming a temporal change of high viscosity to low viscosity during an earthquake  
85 rupture based on a one-body spring slider model with thermal-pressurized slip-  
86 weakening friction. His results in a temporal variation from nucleation phases to  $P$   
87 wave and the amplitude of  $P$  wave, which is associated with the earthquake  
88 magnitude, does not depend on the duration time of the former.

89 As mentioned above, the nucleation process behaves like a transition from quasi-  
90 static slip (without the inertial effect) to (unstable) dynamic motion (with the inertial  
91 effect) when the slip speeds become high enough to make the inertial effect dominate  
92 frictional resistance under some conditions. This assumes that the inertial effect must  
93 be taken into account.

94 In most of studies, both the nucleation phase and the  $P$  wave are assumed to  
95 occur on the same fault. There is an interesting question: Can the nucleation phase  
96 happen on a sub-fault which links to the main fault of an earthquake? In order to  
97 answer this question, in this work I will explore the frictional, viscous, and inertial  
98 effects on the generation of nucleation phase on a fault and then the transition from it  
99 to the  $P$  wave on the other based on a two-body spring-slider model, which is used to  
100 approach an earthquake fault (see Galvanetto, 2002; Turcotte, 1992), by considering  
101 the two sliders to be two segments of an earthquake fault,. The friction force caused  
102 by thermal pressurization is slip-weakening and the viscosity is represented by an  
103 explicit parameter. In addition, it is significant to consider the inertial effect on the  
104 earthquake nucleation because of the existence of transition from quasi-static motion



105 to dynamic ruptures from observations and laboratory experiments. The study on  
106 inertial effect on nucleation phase is rare, even though this effect is implicitly  
107 included in the thermal-pressurized friction used by Brantut et al. (2011). Here, the  
108 inertial effect will be taken into account.

109

## 110 **2 Two-body Spring-slider Model**

111 The two-body spring-slider model (Fig. 2) consists of two sliders of mass  $m_i$  ( $i=1, 2$ )  
112 and three springs. The detailed description of the model can be seen in Wang (2017b)  
113 and only briefly explained here. The equation of motion of the system is

114

$$115 \quad m_1 d^2 u_1 / dt^2 = K(u_2 - u_1) - L_1(u_1 - v_p t) - F_1(u_1) - \Phi(v_1) \quad (1a)$$

116

$$117 \quad m_2 d^2 u_2 / dt^2 = K(u_1 - u_2) - L_2(u_2 - v_p t) - F_2(u_2) - \Phi(v_2). \quad (1b)$$

118

119 The  $u_i$  ( $i=1, 2$ ) is the displacement of the slider measured from its initial equilibrium  
120 position along the x-axis. The  $K$  is the strength of the coil spring between two sliders  
121 and the  $L_i$  ( $i=1, 2$ ) is the strength of the leaf spring to yield the driving force on the  
122  $i$ -th slider from a moving plate with a constant speed  $v_p$ . Considering the two sliders  
123 to be two segments of a single earthquake fault, the coupling between the moving  
124 plate and a slider could be equal for the two sliders, thus giving  $L_1 = L_2 = L$ .  $F_i(u_i)$  ( $i=1,$   
125  $2$ ) is the frictional force on the  $i$ -th slider. Wang (2013) took  $F(u) = F_o \exp(-u/u_c)$ ,  
126 where  $F_o$  and  $u_c$  are, respectively, the static friction force and characteristic slip  
127 displacement, to study earthquake dynamics. This friction force is slip-weakening and  
128 caused by the adiabatic-undrained-deformation (AUD)-type thermal pressurization  
129 (Rice, 2006). An example of the variations of  $F(u)$  versus  $u$  for  $F_o = 1$  N and  $u_c = 0.1$ ,



130 0.3, 0.5, 0.7, and 0.9 m is displayed in Fig. 3.  $F(u)$  decreases with increasing  $u$ , and  
 131 the decreases rate is higher for smaller  $u_c$  than for larger  $u_c$ . This indicates that the  
 132 force drop decreases with increasing  $u_c$  for the same final displacement. The  $\Phi(v_i)$ ,  
 133 where  $v_i=du/dt$  is the particle velocity, is a velocity-dependent viscous force.  
 134 According to Stokes' law, Wang (2016) suggested the viscous force to be  $\Phi=Cv$ ,  
 135 where  $C=6\pi R\upsilon$  (with a unit of  $N(m/s)^{-1}$ ) is the damping coefficient of a sphere of  
 136 radius  $R$  in a fluid of viscosity  $\upsilon$  (Kittel et al. 1968). The two sliders rest in an  
 137 equilibrium state at time  $t=0$ . Note that this model addresses only the strike-slip  
 138 component and, thus, cannot completely represent earthquake ruptures, which also  
 139 consist of transpressive components. Nevertheless, simulation results of this model  
 140 can still provide significant information on earthquake ruptures.

141 Substituting the friction and viscous laws into Equation (1) leads to

142

$$143 \quad m_1 d^2 u_1 / dt^2 = K(u_2 - u_1) - L(u_1 - v_p t) - F_{o1} \exp(-u_1 / u_{c1}) - C_1 du_1 / dt \quad (2a)$$

144

$$145 \quad m_2 d^2 u_2 / dt^2 = K(u_1 - u_2) - L(u_2 - v_p t) - F_{o2} \exp(-u_2 / u_{c2}) - C_2 du_2 / dt \quad (2b)$$

146

147 To deal with the problem easily, it is usual to normalize Equation (2) based on the  
 148 normalization parameters. Wang (1995) defined the stiffness ratio,  $s$ , to be the ratio of  
 149  $K$  to  $L$ , i.e.,  $s=K/L$ . Wang (2017b) defined the normalization parameters for Equation  
 150 (2). However, in his study he took  $m_1=m_2$ , and thus he did not consider the cases with  
 151 different values of  $m_1$  and  $m_2$ . While, in this study  $m_2$  could be larger than  $m_1$  for  
 152 showing the inertial effect. Hence, the parameters normalizing Equation (2) are:

$$153 \quad m_1=m, \quad m_2=\mu m, \quad F_{o1}=F_o, \quad F_{o2}=\phi F_o, \quad D_o=F_o/L, \quad \omega_{o1}=\omega_o=(L/m)^{1/2}, \quad \omega_{o2}=\mu^{-1/2}\omega_o, \quad \tau=\omega_o t,$$

$$154 \quad u_{c1}=u_c, \quad u_{c2}=\psi u_c, \quad U_{c1}=u_c/D_o, \quad U_{c2}=\psi u_c/D_o, \quad f_{o1}=f_o=F_o/D_o, \quad f_{o2}=\phi f_o, \quad \eta_1=C_1\omega_o/L,$$



155  $\eta_2 = C_2 \mu^{-1/2} \omega_0 / L$ ,  $\gamma = \eta_2 / \eta_1$ , and  $V_p = v_p / D_o \omega_0$ . Defining  $U_i = u_i / D_o$  and  $V_i = dU_i / d\tau$  leads to  
 156  $du_i / dt = [F_o / (mL)^{1/2}] dU_i / d\tau$  and  $d^2 u_i / dt^2 = (F_o / m) d^2 U_i / d\tau^2$ . Inserting these normalization  
 157 parameters with  $f_0 = 1$  into Equation (2) results in:

158

$$159 \quad d^2 U_1 / d\tau^2 = s(U_2 - U_1) - (U_1 - V_p \tau) \exp(-U_1 / U_{c1}) - \eta_1 dU_1 / d\tau \quad (3a)$$

160

$$161 \quad d^2 U_2 / d\tau^2 = [s(U_1 - U_2) - (U_2 - V_p \tau) \exp(-U_2 / U_{c2}) - \eta_2 dU_2 / d\tau] / \mu. \quad (3b)$$

162

163 Let  $y_1 = U_1$ ,  $y_2 = U_2$ ,  $y_3 = dU_1 / d\tau$ , and  $y_4 = dU_2 / d\tau$ . Equation (3) can be re-written  
 164 as four first-order differential equations:

165

$$166 \quad dy_1 / d\tau = y_3 \quad (4a)$$

167

$$168 \quad dy_2 / d\tau = y_4 \quad (4b)$$

169

$$170 \quad dy_3 / d\tau = -(s+1)y_1 + sy_2 - \exp(-y_1 / U_{c1}) - \eta_1 y_3 + V_p \tau \quad (4c)$$

171

$$172 \quad dy_4 / d\tau = [sy_1 - (s+1)y_2 - \exp(-y_2 / U_{c2}) - \gamma \eta_1 y_4 + V_p \tau] / \mu. \quad (4d)$$

173

174 Since it is difficult to analytically solve Equation (4), only numerical simulations  
 175 using the fourth-order Runge-Kutta method (see Press et al., 1986) is performed in  
 176 this study. Note that the sliders are restricted to move only along the positive direction,  
 177 that is,  $V_i \geq 0$  and  $U_i \geq 0$  ( $i=1, 2$ ).

178

### 179 3 Numerical Simulations



180 Before performing numerical simulations, it is necessary to consider the acceptable  
181 values of model parameters. Strong coupling can make the two sliders move almost  
182 simultaneously. Hence, in order to allow independent motion for each slider, the value  
183 of  $s$  should be small. Numerical tests (Wang, 2017b) show weak coupling as  $s < 5$  and  
184 strong coupling as  $s \geq 5$  for a two-body spring-slider system. Hence,  $s < 5$  is considered  
185 in this study. In general,  $v_p$  is  $\sim 10^{-9}$  m/s and thus  $V_p$  is  $\sim 10^{-9}$  when  $D_o \omega_o$  is an order of  
186 magnitude of 1 m/sec. Simulation results could be influenced by using various time  
187 steps,  $\delta\tau$ . Practical tests suggest that simulation results show numerical stability when  
188  $\delta\tau < 0.05$ . The time step is taken to be  $\delta\tau = 0.02$  hereafter. When  $V_p \tau = \exp(-y_l/U_{c1})$  on  
189 slider 1 from Equation (4c), the force exerted from the moving plate is just equal to  $f_{oi}$ .  
190 Although in principle slider 1 can start to move under this condition, in practice the  
191 computation cannot go ahead because all values are zero. An initial force,  $\mathcal{F}$ , is  
192 necessary to kick off slider 1. Note that the value of  $\mathcal{F}$  can affect the computational  
193 results (Carlson et al., 1991). A very small value of  $\mathcal{F}$  cannot enforce slider 1 to move;  
194 while a large one will dominate the whole computation process. Numerical tests show  
195 that  $\mathcal{F} = 10^{-3}$  is appropriate for numerical simulations.

196 Numerical simulations are made under various values of model parameters for  
197 showing the effects caused by seismic coupling, friction, viscosity, and inertial effect.  
198 Simulation results are displayed in Figures 4–10 which include the time variations in  
199  $V/V_{max}$  (in the left-hand-side panels) and  $U/U_{max}$  (in the right-hand-side panels).

200 The results for the effect due to seismic coupling are displayed in Fig. 4 where  
201 the values of  $s$  are: (a) for  $s = 0.06$ , (b) for  $s = 0.12$ , (c) for  $s = 0.30$ , and (d) for  $s = 0.48$   
202 when  $f_{o1} = 1.0$  and  $f_{o2} = 1.0$  (with  $\phi = 1$ ),  $U_{c1} = 0.5$  and  $U_{c2} = 0.5$  (with  $\psi = 1$ ), and  $\eta_1 = 0$  and  
203  $\eta_2 = 0$  (with  $\gamma = 1$ ). First, it is necessary to examine the lower-bound value of  $s$  for  
204 yielding strong enough coupling between the two sliders. Numerical tests exhibit that



205 slider 2 cannot move for  $s < 0.06$  when other model parameters are equal on the two  
206 sliders. Hence,  $s = 0.06$  is almost the lower bound of seismic coupling for most of  
207 simulations. On the other hand, numerical tests suggest that when  $s > 0.48$ , the solid  
208 and dashed lines are coincided. This means that large  $s$  having strong seismic  
209 coupling leads to almost simultaneous motions of the two sliders. Hence, the value of  
210  $s$  is taken to be 0.48 in Figs. 5–7, and 9 to explore which factor can separate the  
211 motions of the two sliders.

212 Figures 5–8 display the results due to different values of viscosity on the two  
213 sliders when other parameters are fixed: (a) for  $\gamma = 0.00$  (i.e.,  $\eta_2 = 0$ ), (b) for  $\gamma = 0.01$  (i.e.,  
214  $\eta_2 = 0.1$ ), (c) for  $\gamma = 0.05$  (i.e.,  $\eta_2 = 0.5$ ), and (d) for  $\gamma = 0.10$  (i.e.,  $\eta_2 = 1$ ) when  $\eta_1 = 10$ . In  
215 Fig. 5 the values of other model parameters are  $\mu = 1$ ,  $\eta_1 = 10$ ,  $s = 0.48$ ,  $f_{o1} = 1.0$  and  
216  $f_{o2} = 1.0$  (with  $\phi = 1$ ), and  $U_{c1} = 0.5$  and  $U_{c2} = 0.5$  (with  $\psi = 1$ ). The figure displays the  
217 presence of the  $P$  wave on slider 2. Numerical tests reveal that the  $P$  wave on slider 2  
218 cannot be generated especially for  $\gamma \geq 0.05$  when  $\eta_1 > 70$ , and the solutions are just like  
219 Figure 4 when  $\eta_1 < 5$ . Hence,  $\eta_1$  is taken to be 10 in Figs. 6–10. The simulation results  
220 to exhibit the effect due to different static friction strengths on the two sliders, are  
221 displayed in Fig. 6, where the values of other model parameters are  $\mu = 1$ ,  $\eta_1 = 10$ ,  
222  $s = 0.48$ ,  $f_{o1} = 1.0$  and  $f_{o2} = 1.1$  (with  $\phi = 1.1$ ), and  $U_{c1} = 0.5$  and  $U_{c2} = 0.5$  (with  $\psi = 1$ ). The  
223 figure exhibits the presence of a nucleation phase on slider 1. Numerical tests exhibit  
224 that when  $\phi > 1.15$ , the  $P$  wave on slider 2 cannot be generated. Hence,  $\phi$  is taken to be  
225 1.1 in Figs. 7–10. The simulation results to exhibit the effect due to different  
226 characteristic displacements on the two sliders are displayed in Fig. 7, where the  
227 values of other model parameters are  $\mu = 1$ ,  $\eta_1 = 10$ ,  $s = 0.48$ ,  $f_{o1} = 1.0$  and  $f_{o2} = 1.1$  (with  
228  $\phi = 1.1$ ), and  $U_{c1} = 0.5$  and  $U_{c2} = 0.1$  (with  $\psi = 0.2$ ). The figure shows the presence of a  
229 nucleation phase on slider 1. Numerical tests exhibit that when  $U_{c1} > 0.5$ , the  $P$  wave



230 on slider 2 cannot be generated. Hence,  $U_{c1}$  is taken to be 0.5 in Figs. 8–10. In order  
231 to consider weaker seismic coupling on the simulated waveforms, smaller  $s$  is taken  
232 into account. Numerical tests exhibit that when  $s < 0.17$ , the  $P$  wave on slider 2 cannot  
233 be generated. Hence,  $s$  is also taken to be 0.17 in Fig. 8 where the values of other  
234 model parameters are  $\mu=1$ ,  $\eta_1=10$ ,  $s=0.17$ ,  $f_{o1}=1.0$  and  $f_{o2}=1.1$  (with  $\phi=1.1$ ), and  
235  $U_{c1}=0.5$  and  $U_{c2}=0.1$  (with  $\psi=0.2$ ).

236 Figures 9 and 10 display the results for the inertial effect due to different masses  
237 of the two sliders: (a) for  $\mu=1$ , (b) for  $\mu=5$ , (c) for  $\mu=10$ , and (d) for  $\mu=30$  when  $\eta_1=10$ ,  
238  $f_{o1}=1.0$  and  $f_{o2}=1.1$  (with  $\phi=1.1$ ),  $U_{c1}=0.5$  and  $U_{c2}=0.1$  (with  $\psi=0.2$ ), and  $\eta_1=10$  and  
239  $\eta_2=0$  (with  $\gamma=0$ ). The main difference between the two figures is the use of different  
240 values of seismic coupling:  $s=0.48$  in Fig. 9 and  $s=0.17$  in Fig. 10.

241 In the panels of Figs. 4–10, the simulation results for slider 1 and slider 2 are  
242 represented, respectively, by a solid line and a dotted line. Numerical results show that  
243 the values of  $V_{max}$  and  $U_{max}$  are: 0.456 and 1.355, respectively, in Fig. 4; 0.142 and  
244 0.798, respectively, in Fig. 5; 0.226 and 0.766, respectively, in Fig. 6; 0.781 and 1.403,  
245 respectively, in Fig. 7; 0.903 and 1.778, respectively, in Fig. 8; 0.781 and 1.505,  
246 respectively, in Fig. 9; and 0.903 and 1.790, respectively, in Fig. 10.

247

## 248 4 Discussion

### 249 4.1 Seismic Coupling Effect

250 Figure 4 shows the simulation results when  $s=0.06$ , 0.12, 0.30, and 0.48 (upside  
251 down). In the left-hand-side panels for  $V/V_{max}$ , the dashed line separates from the  
252 solid line for small  $s$ , while the two lines are almost coincided for large  $s$ . This reflects  
253 the fact that seismic coupling between the two sliders increases with  $s$ . Meanwhile,  
254 the peak amplitude is larger at slider 2 than at slider 1. This is reasonable due to the



255 directivity effect because the system moves from slider 1 to slider 2. However, this  
256 figure does not exhibit the existence of long-period nucleation phase. Hence, seismic  
257 coupling is not a significant factor in the generation of nucleation phase. From the  
258 right-hand-side panels for  $U/U_{max}$ , we can also obtain the same conclusion as  
259 mentioned above. In addition, the final displacements on the two sliders are almost  
260 equal.

261

#### 262 4.2 Viscous Effect

263 Simulation results based on a one-body spring-slider model by Wang (2017a) show  
264 that a change of viscosity from a larger value to a small one in two time stages during  
265 slippage yields the nucleation phase and the  $P$  wave, respectively, in the first and  
266 second stages. Hence, in Fig. 5 the value of  $\eta_1$  is set to be 10 and  $\eta_2$  varies from 0 to 1  
267 or  $\gamma$  varies from 0.0 to 0.1 for  $s=0.48$  when the values of other parameters are the  
268 same as those in Fig. 4. The left-hand-side panels of Fig. 5 exhibit the presence of a  
269 short-time nucleation phase plus a smaller event on slider 1 and a larger event with a  
270  $P$  wave on slider 2. Hence, there are two sub-events during the whole rupture process.  
271 The peak velocity of slider 2 decreases with increasing  $\gamma$ , yet not for slider 1. The  
272 peak velocity appears earlier on slider 2 than on slider 1. The occurrence time of the  
273 peak velocity of slider 2 slightly increases with  $\gamma$ . In addition, there are few events  
274 with low peak velocities after the main one on slider 2, and the number of small  
275 events decreases with increasing  $\gamma$ .

276 The predominant period and the peak velocity of slider 1 are, respectively, longer  
277 and smaller than those of slider 2. Of course, the differences decrease with increasing  
278  $\gamma$  or  $\eta_2$ . Compared with Fig. 4, the predominant periods for the two sliders in Fig. 5  
279 become longer due to the viscous effect. From Equation (1), the (dimensionless)



280 natural period is  $T_{o1}=T_o=2\pi(m/L)^{1/2}$  for slider 1 and  $T_{o2}=2\pi(\mu m/L)^{1/2}=\mu^{1/2}T_o$  for  
281 slider 2 when the two sliders are not linked together and friction and viscosity are  
282 both absent. When the two sliders are linked together, the natural period of each slider  
283 must be slightly different from  $T_{o1}$  or  $T_{o2}$ . When viscosity is included, the natural  
284 period is  $T_1=T_{o1}/(1-C_1^2/4mL)^{1/2}=T_o/(1-\eta_1^2/4)^{1/2}$  for slider 1 and  
285  $T_2=T_{o2}/(1-C_2^2/4\mu mL)^{1/2}=\mu^{1/2}T_o/(1-\eta_2^2/4)^{1/2}$  for slider 2. Obviously, viscosity  
286 increases the natural period of oscillations of each slider and also depresses the peak  
287 velocity. The ratio of  $T_2$  to  $T_1$  is:

288

$$289 \quad T_2/T_1=[\mu(4-\eta_1^2)/(4-\eta_2^2)]^{1/2}=[\mu(4-\eta_1^2)/(4-\gamma_1^2)]^{1/2}. \quad (5)$$

290

291 Equation (5) shows that when  $\mu>1$  and  $\gamma>1$ , we have  $T_2>T_1$ . When  $\eta_2$  approaches 2,  
292  $T_2$  becomes infinity. Hence,  $\eta_2=2$  is an upper bound of generating a normal  $P$  wave.  
293 The left-hand-side panels of Fig. 5 exhibits an increase in  $T_2$  with  $\eta_2$  or  $\gamma$ .

294 In the right-hand-side panels of Fig. 5, the displacement of slider 2 (displayed by  
295 a dashed line) first increases more rapidly than that of slider 1 (shown by a solid line)  
296 and finally two lines merge together, thus exhibiting the same final displacement on  
297 the two sliders.

298 Although we can see the existence of long-period waveform on slider 1 in Fig. 5,  
299 its peak velocity comes after that of a short-period  $P$  wave on slider 2. This does not  
300 exhibit transition from quasi-static motions to dynamic ruptures as shown from  
301 observations, and thus the whole waveform on slider 1 cannot be classified to be the  
302 nucleation phase. Hence, it is assumed that different values of viscosity coefficients  
303 on the two sliders are not the unique factor to yield the nucleation phase for the  
304 two-body model, and thus the differences in other model parameters between the two



305 sliders must be taken into account.

306

307 4.3 Frictional Effect

308 The frictional effect includes two components: the static friction forces or the frictional

309 strength (denoted by  $f_{o1}$  and  $f_{o2}$  on slider 1 and slider 2, respectively) and the

310 characteristic displacements of friction law (represented by  $U_{c1}$  and  $U_{c2}$  on slider 1

311 and slider 2, respectively). First, we consider different values of  $s$ ,  $f_{o1}$ , and  $f_{o2}$ .

312 Simulation results are displayed in Fig. 6 where static friction forces are  $f_{o1}=1.0$  and

313  $f_{o2}=1.1$  (with  $\phi=1.1$ ) when other values of model parameters are the same as those in

314 Fig. 5, i.e.,  $s=0.48$ ,  $U_{c1}=0.5$  and  $U_{c2}=0.5$  (with  $\psi=1$ ), and (a) for  $\gamma=0.00$  or  $\eta_2=0$ , (b)

315 for  $\gamma=0.01$  or  $\eta_2=0.1$ , (c) for  $\gamma=0.05$  or  $\eta_2=0.5$ , and (d) for  $\gamma=0.10$  or  $\eta_2=1$  when  $\eta_1=10$ .

316 The left-hand-side panels show the presence of a very long-duration nucleation phase

317 on slider 1 in the front of the  $P$  wave on slider 2. After slider 2 stopped motion, slider

318 1 still moves and its peak velocity comes after that of slider 2. The occurrence time of

319 the peak velocity slightly increases with  $\gamma$ . Although a bump appears in the waveform

320 of slider 1, its peak velocity is much smaller than that of slider 2. Hence, unlike Fig. 5

321 there is almost only one event in the whole rupture process in Fig. 6. Meanwhile, the

322 maximum value of peak velocity of Fig. 6 is higher than that of Fig. 5. In the

323 right-hand-side panels of Fig. 6, the displacements of slider 1 (displayed by a solid

324 line) and slider 2 (displayed by a dashed line) appear almost simultaneously and

325 increase with time. The final displacement is higher on slider 2 than on slider 1, and

326 the difference between the two final displacements decreases with increasing  $\gamma$ .

327 Tal et al. (2018) who studied numerically the effects of fault roughness with

328 amplitude of  $b_r$  on the nucleation process of earthquakes in the presence of a rat- and

329 state-dependent friction law. The roughness can yields local barriers and makes the



330 nucleation process complicated. They also found an increase in nucleation length with  
331  $b_r$ . Considering a broad weak zone with a locally strong asperity on a fault plane,  
332 Shibazaki and Matsu'ura (1995) found that in the dynamic rupture of the asperity,  
333 there are aseismic slip and foreshock or pre-event, depending on the peak stress of the  
334 asperity, preceding the main rupture and the rupture of the asperity accelerates the  
335 nucleation of main rupture. This study indicates the influence of heterogeneous  
336 friction strengths on the generation of nucleation phase. Schmitt et al. (2015)  
337 considered the importance of time-dependent stress heterogeneity on nucleation.  
338 Although this factor is not taken into account in this study, the present study for  
339 different values of  $\phi$  on the two sliders seems able to meet the results obtained by the  
340 three groups.

341 Secondly, we consider different values of  $U_{c1}$  and  $U_{c2}$ . Simulation results are  
342 displayed in Fig. 7 where the values of characteristic displacements are  $U_{c1}=0.5$  and  
343  $U_{c2}=0.1$  (with  $\psi=0.2$ ) and the values of other model parameters are the same as those  
344 in Fig. 6, i.e.,  $s=0.48$ ,  $f_{o1}=1.0$  and  $f_{o2}=1.1$  (with  $\phi=1.1$ ), and (a) for  $\gamma=0.00$  or  $\eta_2=0$ , (b)  
345 for  $\gamma=0.01$  or  $\eta_2=0.1$ , (c) for  $\gamma=0.05$  or  $\eta_2=0.5$ , and (d) for  $\gamma=0.10$  or  $\eta_2=1$  when  $\eta_1=10$ .  
346 Like Fig. 6, the left-hand-side panels show the existence of a very long-duration  
347 nucleation phase on slider 1 in the front of the  $P$  wave on slider 2. After slider 2  
348 stopped motion, slider 1 still moves and its peak velocity comes after that of slider 2.  
349 The occurrence time of the peak velocity slightly increases with  $\gamma$ . The maximum  
350 value of peak velocity of Fig. 7 is higher than that of Fig. 6. In addition, the  
351 predominant period of  $P$  wave on slider 2 is shorter in Fig. 7 than in Fig. 6. This  
352 might be due to a faster drop of friction force in Fig. 7 with shorter  $U_{c2}$  than in Fig. 6  
353 with longer  $U_{c2}$ . Although a peak velocity appears in the waveform of slider 1, its  
354 amplitude is very much smaller than that of slider 2. Hence, unlike Fig. 5 there is



355 almost only one event in the whole rupture process of Fig. 7. In the right-hand-side  
356 panels of Fig. 7, the displacement of slider 1 (displayed by a solid line) first appears  
357 and increases with time; while the displacement of slider 2 (displayed by a dashed line)  
358 suddenly appears for a while after slider 1 moves and then jumps to its peak value in a  
359 short time. The final displacement is higher on slider 2 than on slider 1, and the  
360 difference between the two final displacements decreases with increasing  $\gamma$ .

361 Using an infinite elastic model with a slip-dependent friction, Ionescu and  
362 Campillo (1999) found the influence of the shape of the friction law and fault  
363 finiteness on the duration of nucleation phase and the duration varies when the fault  
364 length has the order of the characteristic length of the friction law. The present study  
365 is essentially consistent with their results.

366 Thirdly, it is necessary to consider the effect on the simulations due to weak  
367 seismic coupling (now  $s=0.17$ ) between the two sliders when the values of other  
368 model parameters are the same as those in Fig. 7, i.e.,  $f_{o1}=1.0$  and  $f_{o2}=1.1$  (with  $\phi=1.1$ ),  
369 and  $U_{c1}=0.5$  and  $U_{c2}=0.1$  (with  $\psi=0.2$ ), and (a) for  $\gamma=0.00$  or  $\eta_2=0$ , (b) for  $\gamma=0.01$  or  
370  $\eta_2=0.1$ , (c) for  $\gamma=0.05$  or  $\eta_2=0.5$ , and (d) for  $\gamma=0.10$  or  $\eta_2=1$  when  $\eta_1=10$ . Simulation  
371 results are displayed in Fig. 8. Like Figs. 6 and 7, there is a very long-duration  
372 nucleation phase on slider 1 in the front of the  $P$  wave on slider 2. After slider 2  
373 stopped motion, slider 1 still moves and its peak velocity comes after that on slider 2.  
374 The peak velocity of slider 1 appears much later than that in Fig. 7. This might be due  
375 to a fact that it needs a longer time to trigger slider 2 due to weak coupling between  
376 the two sliders in Fig. 8. Meanwhile, the occurrence time of the peak velocity on  
377 slider 2 slightly increases with  $\gamma$ . From the values of peak velocity as mentioned  
378 above, the maximum value of peak velocity in Fig. 8 is higher than that in Fig. 7. This  
379 indicates that weaker coupling between two sliders can yield a higher peak velocity



380 than stronger coupling. In addition, the predominant period of  $P$  wave on slider 2 is  
381 shorter in Fig. 8 than in Fig. 7. Although a peak velocity appears in the waveform of  
382 slider 1, its amplitude is very much smaller than that of slider 2. Unlike Fig. 5 there is  
383 almost only one event in the whole rupture process of Fig. 8.

384 In the right-hand-side panels of Fig. 8, the displacement of slider 1 (displayed by  
385 a solid line) first appears and increases with time; while the displacement of slider 2  
386 (displayed by a dashed line) suddenly appears for a while after slider 1 moves and  
387 then jumps to its peak value in a short time span. The final displacement is higher on  
388 slider 2 than on slider 1, and the difference between the two final displacements  
389 decreases with increasing  $\gamma$ .

390

#### 391 4.4 Inertial Effect

392 The inertial effect (represented by  $\mu$ ) on the earthquake nucleation is made for  
393 different masses of the two sliders, i.e.,  $\mu > 1$ . Simulation results are displayed in Fig. 9  
394 with  $s=0.48$  and in Fig. 10 with  $s=0.48$ . In the two figures, the values of  $\mu$  are: (a) for  
395  $\mu=1$ , (b) for  $\mu=5$ , (c) for  $\mu=10$ , and (d) for  $\mu=30$  when  $s=0.48$ ,  $f_{o1}=1.0$  and  $f_{o2}=1.1$   
396 (with  $\phi=1.1$ ),  $U_{c1}=0.5$  and  $U_{c2}=0.1$  (with  $\psi=0.2$ ), and  $\eta_1=10$  and  $\eta_2=0$  (with  $\gamma=0$ ).

397 Like Figs. 6–8, Figs. 9 and 10 show the existence of very long-duration nucleation  
398 phases on slider 1 in the front of the  $P$  wave on slider 2. After slider 2 stopped moving,  
399 slider 1 still moves and its peak velocity comes after that on slider 2. The occurrence  
400 times of the peak velocity of both sliders 1 and 2 in Figs. 9 and 10 increase with  $\mu$  and  
401 are almost similar to those in Figs. 7 and 8, respectively. The occurrence times of the  
402 peak velocity in Fig. 10 are longer than those in Fig. 9. This might be due to a fact  
403 that a longer time is needed to trigger slider 2 due to weak coupling between the two  
404 sliders in Fig. 10. Meanwhile, the predominant periods of the  $P$  wave on slider 2



405 increases with  $\mu$  as expected. From the values of peak velocity as mentioned above,  
406 the maximum value of peak velocity of Fig. 10 is higher than that of Fig. 9. This  
407 indicates that that weaker coupling between two sliders can yield a higher peak  
408 velocity on slider 2 than stronger coupling. In addition, the peak velocity on slider 1 is  
409 lower than that on slider 2 and decreases with  $\mu$  especially for small  $s$ . Although a  
410 peak velocity appears in the waveform of slider 1 in Figs. 9 and 10, its amplitude is  
411 much smaller than that of slider 1. Unlike Figure 5, there is almost only one event in  
412 the rupture process in Figs. 9 and 10.

413 In Figs. 9 and 10, the peak velocity of  $P$  wave decreases with increasing  $\mu$ .  
414 Numerical tests exhibit that the  $P$  wave almost becomes a nucleation phase on slider 2  
415 when  $\mu > 30$ . In the other word, the nucleation phase on slider 1 cannot trigger the  $P$   
416 wave on slider 2 when the mass of the latter is 30 times larger than that of the former.  
417 When the densities and fault widths of the two sliders are equal, the fault length of  
418 slider 2 is 30 times longer than that of slider 1 when  $\mu = 30$ . Since the present model is  
419 a strike-slip (SS) one, the empirical relationship of earthquake magnitude,  $M$ , versus  
420 fault length,  $L$ , for the SS events is:  $M = (5.16 \pm 0.13) + (1.12 \pm 0.08) \log(L)$  (Wells and  
421 Coppersmith, 1994). When  $\mu = 30$  or  $L_2 = 30L_1$ , the related magnitudes are  $M_1$  for slider  
422 1 and  $M_1 + 1.65$  for slider 2. This means that a nucleation phase with a magnitude of  $M$   
423 cannot trigger an earthquake with a magnitude of  $M + 1.65$ .

424 In the right-hand-side panels of Figs. 9 and 10, the displacement of slider 1  
425 (displayed by a solid line) first appears and increases with time; while the  
426 displacement of slider 2 (displayed by a dashed line) suddenly appears for a while  
427 after slider 1 moves and then jumps to its peak value in a short time span. The  
428 difference in final displacement between the two sliders slightly increases with  $\mu$  and



429 is bigger for small  $s$  than for large  $s$ . The phenomenon that the final displacement of  
430 slider 1 is lower than that of slider 2 might be due to a fact that the force drop on  
431 slider 2 is higher than that on slider 1.

432

#### 433 4.5 Some Comparisons with Other Studies

434 Numerical simulations of this study exhibit that the ratios  $\gamma=\eta_2/\eta_1$ ,  $\phi=f_{o2}/f_{o1}$ ,  
435  $\psi=U_{c2}/U_{c1}$ , and  $\mu=m_2/m_1$  are four important factors in influencing the earthquake  
436 rupture processes including the generation of nucleation phase, yet the seismic  
437 coupling  $s$  is a minor one. Except for the cases with equal values on the two sliders for  
438 the four ratios, the nucleation phase happens on slider 1 and the  $P$  wave appears on  
439 slider 2. When  $\gamma>1$ ,  $\phi=1$ ,  $\psi=1$ , and  $\mu=1$ , there is only a very short-duration nucleation  
440 phase and the  $P$  wave appears very soon after the generation of nucleation phase. This  
441 is inconsistent with Figure 1.

442 When  $\gamma>1$ ,  $\phi>1$ ,  $\psi\geq 1$ , and  $\mu=1$ , there is a long-duration nucleation phase on  
443 slider 1, the  $P$  wave appears on slider 2 much later after the generation of nucleation  
444 phase. Although the simulated waveforms are consistent with Fig. 1, the final  
445 displacement of nucleation phase on slider 1 is the same as that of the  $P$  wave on  
446 slider 2. This indicates equal values of total energy on the two sliders. It is  
447 questionable, because the energy of nucleation phase is lower than that of the  
448 mainshock from observations.

449 When  $\gamma>1$ ,  $\phi>1$ ,  $\psi<1$ , and  $\mu=1$ , the final displacement of nucleation phase is  
450 smaller than that of  $P$  wave. The difference in the amplitudes between the  $P$  wave and  
451 nucleation phase decreases with increasing  $s$ , increasing  $\gamma$ , or decreasing  $\psi$ . The  
452 simulated waveforms are consistent with Fig. 1. The results are reasonable, because  
453 the total energy on slider 1 is less than that on slider 2.



454           When  $\gamma > 1$ ,  $\phi > 1$ , and  $\psi < 1$ , the peak velocity of slider 2 decreases with increasing  
455  $\mu$ , and becomes very small when  $\mu > 30$ , even though the final displacement of  
456 nucleation phase is still smaller than that of  $P$  wave. The degree of similarity of  
457 simulated waveforms of these cases (see Figs. 9 and 10) with Fig. 1 decreases with  
458 increasing  $\mu$ . The upper-bound value of  $\mu$  to yield transition from nucleation phase to  
459 the  $P$  wave from observations is 30. Consequently, the optimal conditions for  
460 generating the nucleation phase on slider 1 plus the  $P$  wave on slider 1 as displayed in  
461 Figure 1 and the results from other studies are  $\gamma > 1$ ,  $\phi > 1$ ,  $\psi < 1$ , and  $\mu < 30$ . Of course,  
462 there are upper-bound values for  $\gamma$  and  $\phi$  and a lower-bound value for  $\psi$  as mentioned  
463 in the last section. Note that the upper-bound value of a ratio depends on the values of  
464 other ratios.

465           However, a difference between the present study and previous ones is that the  
466 nucleation phase appears on slider 1 and does not disappear after the presence of  $P$   
467 wave on slider 2. This might be due to a use of a two-body model in this study and uses  
468 of a one-body or 1-D model is taken in others. Meanwhile, the mechanism (including  
469 friction and viscosity) to yield the transition from quasi-static motions to dynamic  
470 ruptures proposed in this study is the same as that in Wang (2017a), yet different from  
471 others who only considered the frictional effect. However, unlike Wang (2017a) the  
472 present simulation results cannot lead to the conclusion that the peak amplitude of  $P$   
473 wave, which is associated with the earthquake magnitude, is independent upon the  
474 duration time of nucleation phase. In addition, the inertial effect was not taken into  
475 account by Wang (2017a).

476           Based on an infinite elastic model with slip-dependent friction, Shibazaki and  
477 Matsu'ura (1992) assumed that the transition process includes three phases: phase-I  
478 for the low quasi-static nucleation, phase-II for the onset of dynamic ruptured with



479 slow rupture growth in the absence of seismic-wave radiation, and phase-III for the  
480 high-speed rupture propagation with seismic-wave radiation. Shibazaki and Matsu'ura  
481 (1993) further found that the accelerating stage from phase-II to phase-III is related to  
482 the presence of nucleation phase in the front of the main  $P$  wave. Their results are  
483 similar to those obtained by Ueda et al. (2014, 2015) and Kawamura et al (2018). The  
484 results of this study and Wang (2017a) only show two stages which are comparable  
485 with the phase-II and phase-II stages proposed by Shibazaki and Matsu'ura (1992,  
486 1993). From the analytic solutions of an infinite elastic model with a slip-dependent  
487 friction, Campillo and Ionescu (1997) expressed how the initiation phase determines  
488 the transition to the  $P$  wave and claimed that the transition is controlled by an  
489 apparent supersonic velocity of the rupture front. However, the present result does not  
490 seem to meet their conclusion. According to an infinite elastic model with rate- and  
491 state-dependent friction, Segall and Rice (2006) divided the weakening processes of  
492 ruptures into the nucleation regime dominated by rate and state frictional weakening  
493 and a transition regime to thermal pressurization. In the present study, the thermal-  
494 pressurized slip-weakening friction is considered during the whole rupture process  
495 and the results show a transition from the nucleation phase with smaller  $f_{o1}$  and  $U_{c1}$   
496 on slider 1 to the  $P$  wave with larger  $f_{o2}$  and  $U_{c2}$  on slider 2. Hence, the present result  
497 could be only partly consistent with their conclusion.

498

## 499 **5 Conclusions**

500 We study the frictional and viscous effects on earthquake nucleation based on a  
501 two-body spring-slider model in the presence of thermal-pressurized slip-dependent  
502 friction and viscosity. The stiffness ratio of the system is the ratio of coil spring  $K$   
503 between two sliders and the leaf spring  $L$  between a slider and the background plate



504 and denoted by  $s=K/L$ . The  $s$  is not a significant factor in generating the nucleation  
505 phase. The masses of the two sliders are  $m_1$  and  $m_2$ , respectively. The frictional and  
506 viscous effects are specified by the static friction force,  $f_o$ , the characteristic  
507 displacement,  $U_c$ , and viscosity coefficient,  $\eta$ , respectively. Simulation results show  
508 that friction and viscosity can both lengthen the natural period of the system and  
509 viscosity increases the duration time of motion of the slider. Higher viscosity causes  
510 lower particle velocities than lower viscosity. The ratios  $\gamma=\eta_2/\eta_1$ ,  $\phi=f_{o2}/f_{o1}$ ,  
511  $\psi=U_{c2}/U_{c1}$ , and  $\mu=m_2/m_1$  are four important factors in influencing the generation of a  
512 nucleation phase. When  $\gamma>1$ ,  $\phi=1$ ,  $\psi=1$ , and  $\mu=1$ , the nucleation phase is generated on  
513 slider 1 and the  $P$  wave appear on slider 2. But, the  $P$  wave appears very soon after  
514 the generation of nucleation phase. When  $\gamma>1$ ,  $\phi>1$ ,  $\psi\geq 1$ , and  $\mu=1$ , the  $P$  wave  
515 appears much lately after the generation of nucleation phase. When  $\psi\geq 1$ , the final  
516 displacement of nucleation phase is almost equal to that of  $P$  wave. When  $\psi<1$ , the  
517 final displacement of nucleation phase is smaller than that of  $P$  wave. The difference  
518 in the amplitudes between the  $P$  wave and nucleation phase decreases when either  $s$  or  
519  $\gamma$  increases and  $\psi$  decreases. The peak velocity of  $P$  wave on slider 2 decays with  
520 increasing  $\mu$ , thus suggesting that the inertial effect is important on the rupture  
521 processes. Consequently, when  $s>0.17$ ,  $\gamma>1$ ,  $1.15>\phi>1$ ,  $\psi<1$ , and  $\mu<30$  simulation  
522 results exhibit the generation of nucleation phase on slider 1 and the formation of  $P$   
523 wave on slider 2. The results are consistent with the observations and suggest the  
524 possibility of generation of nucleation phase on a sub-fault. This answer the question  
525 pointed out in this study.

526

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529

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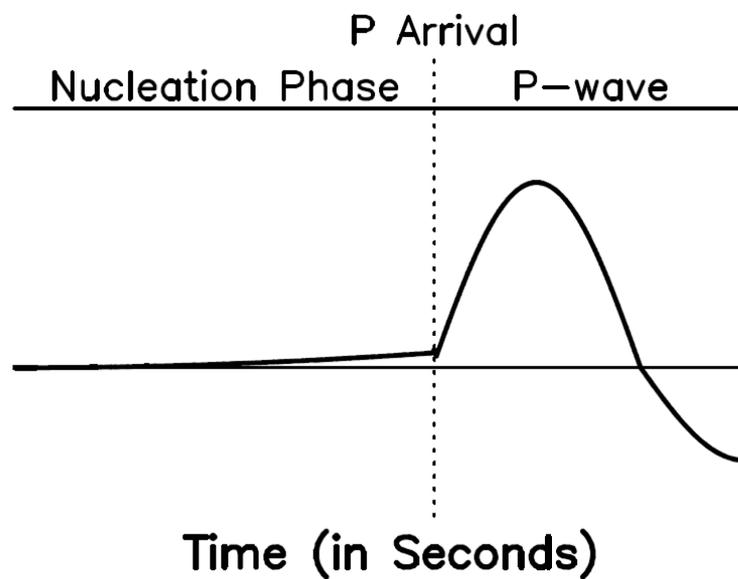
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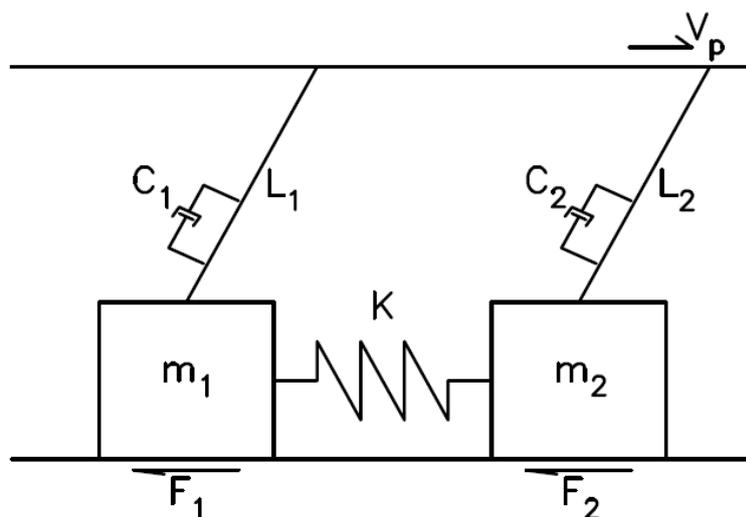
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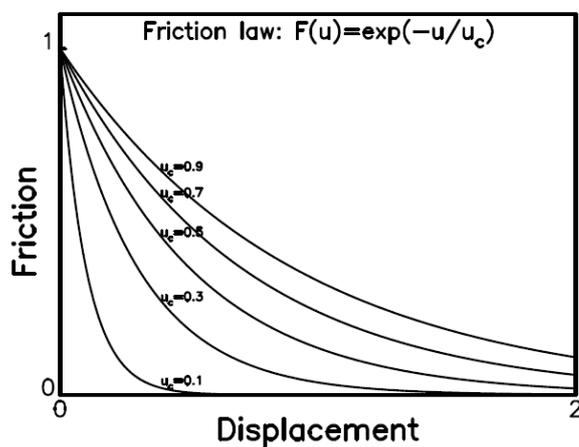
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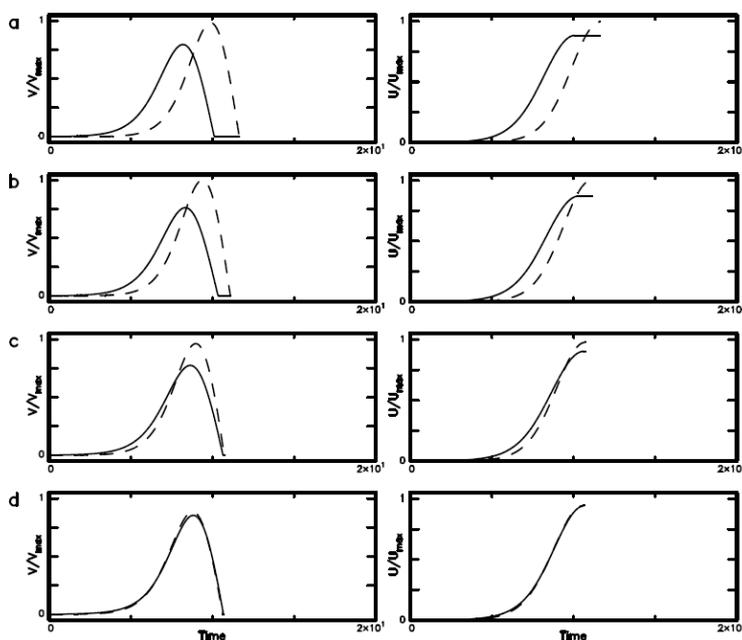
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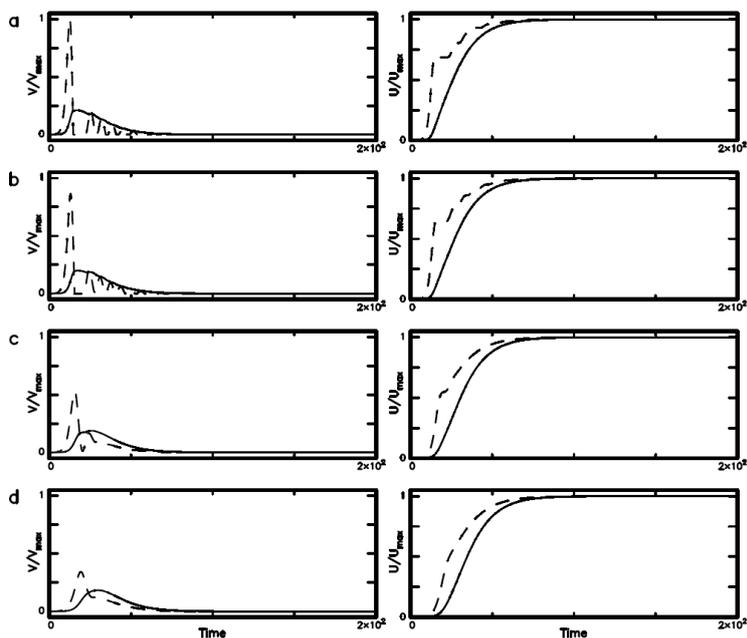
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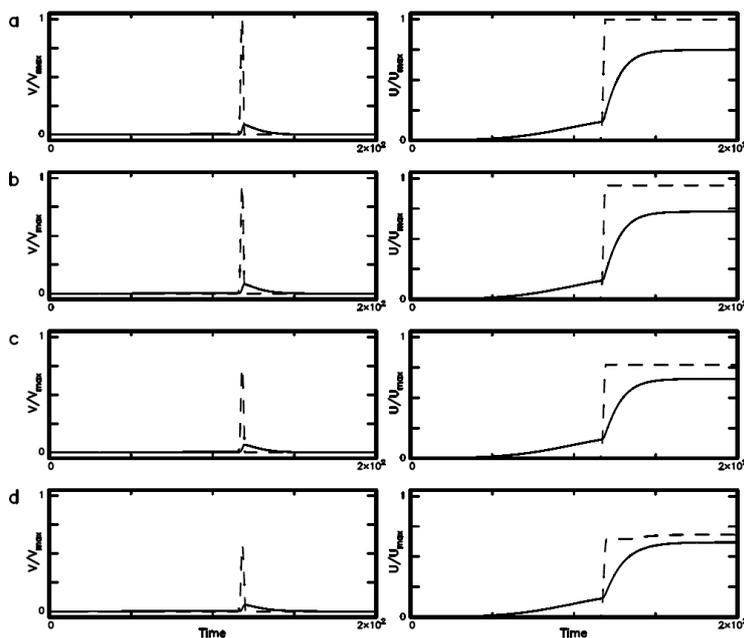
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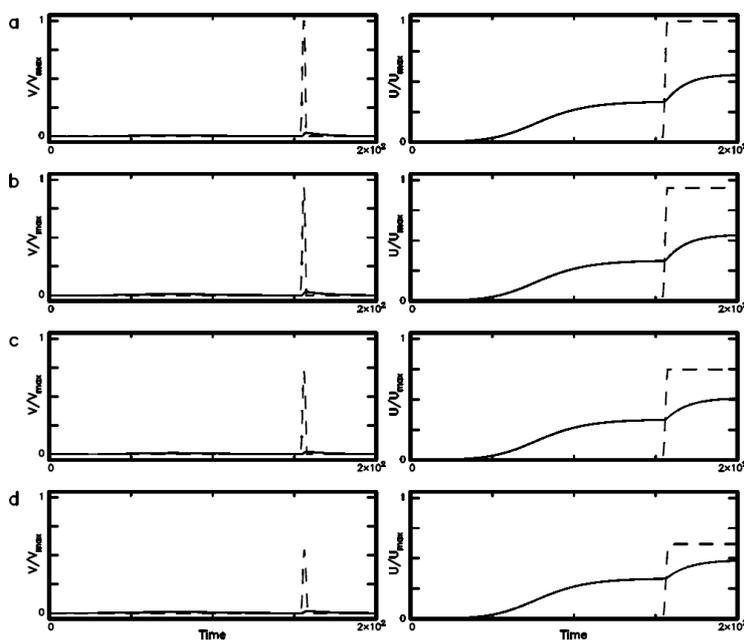
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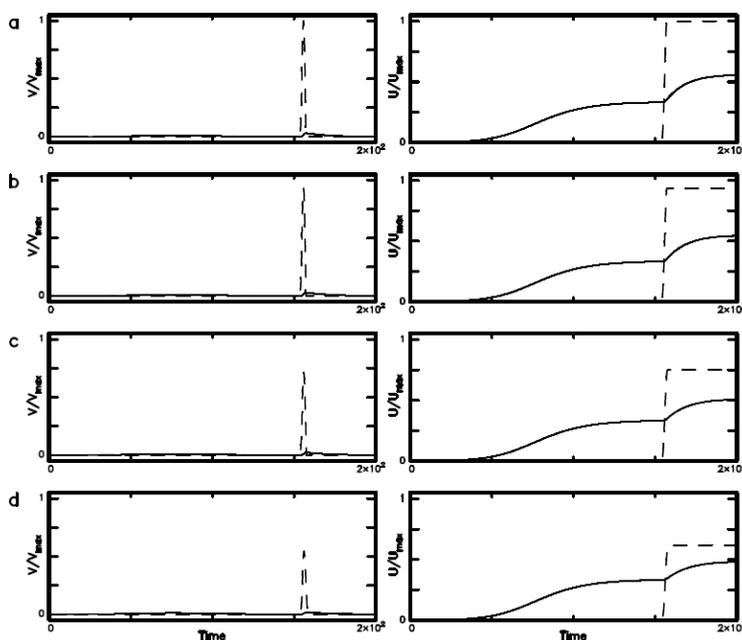
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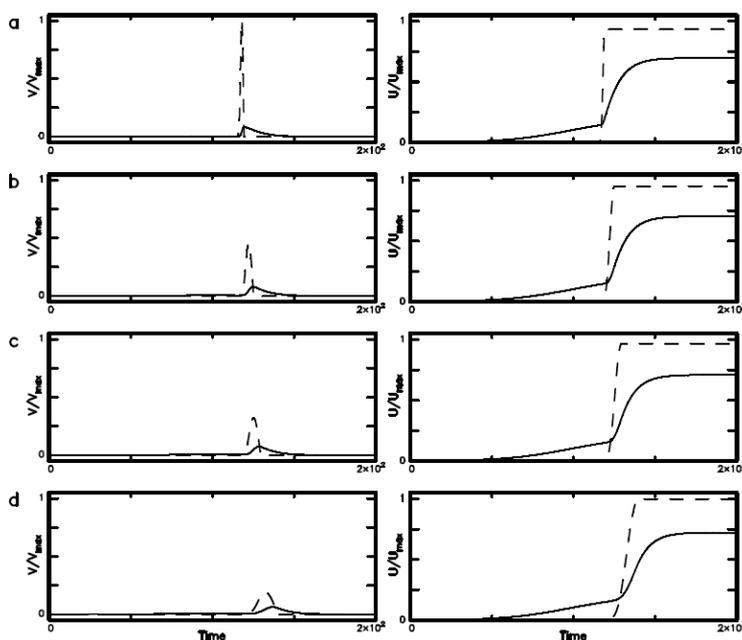
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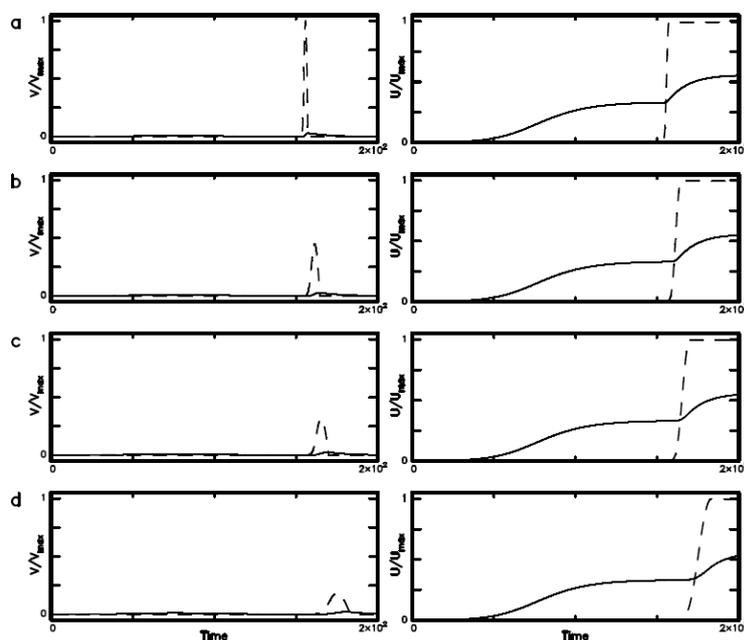
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