Application of fractal models to delineate mineralized zones in the Pulang porphyry copper deposit, Yunnan, Southwest China

Xiaochen Wang\textsuperscript{a}, Qinglin Xia\textsuperscript{a,b,*}, Tongfei Li\textsuperscript{a}, Shuai Leng\textsuperscript{a}, Yanling Li\textsuperscript{a}, Li Kang\textsuperscript{a}, Zhijun Chen\textsuperscript{a}, Lianrong Wu\textsuperscript{c}

\textsuperscript{a}Faculty of Earth Resources, China University of Geosciences, Wuhan 430074, China
\textsuperscript{b}Collaborative Innovation Center for Exploration of Strategic Mineral Resources, Wuhan 430074, China
\textsuperscript{c}Yunnan Diqing Nonferrous MetalCo., Ltd., Shangri-La 674400, China

Abstract

The aim of this study is to delineate and recognize various mineralized zones and barren host rocks based on the surface and subsurface lithogeochemical data utilizing the number-size (N-S), concentration-volume (C-V) and power spectrum-volume (S-V) fractal models in the Pulang porphyry copper deposit, southwest China. The N-S model reveals three mineralized zones characterized by Cu thresholds of 0.28\% and 1.45\%, with zones <0.28\% Cu representing weakly mineralized zones and barren host rocks, with zones 0.28\%-1.45\% Cu representing moderately mineralized zones and zones >1.45\% Cu representing highly mineralized zones. Results obtained by the C-V model depict four geochemical zones defined by Cu thresholds of 0.25\%, 1.48\% and 1.88\%, which represent non-mineralized wall rocks (Cu<0.25\%), weakly mineralized zones (0.25\%-1.48\%), moderately mineralized zones (1.48\%-1.88\%), and highly mineralized zones (Cu>1.88\%). S-V model is used by performing 3D fast Fourier transformation on assay data in the frequency domain. The S-V model reveals three mineralized zones characterized by Cu thresholds of 0.23\% and 1.33\%, with zones of <0.23\% Cu representing leached zone and barren host rocks, with zones of 0.23\%-1.33\% Cu representing the hypogene zones and zones of >1.33\% Cu representing supergene enrichment zones. All the multifractal models indicate that high grade mineralization is situated in the central and southern parts of the ore deposit. Their results are compared with the alteration and mineralogical models resulted from the 3D geological model.
using logratio matrix. The results show that the S-V model gives the best results to identify highly mineralized zones in the deposit. However, the results of C-V model for moderately and weakly mineralized zones are more accurate than the zones obtained from the N-S and S-V model.

Keywords: Fractal; Concentration-volume model (C-V); Number-size model (N-S); Power spectrum-volume model (S-V); Mineralized zone; the Pulang porphyry copper deposit

1. Introduction

The definition and delineation of different mineralized zones and non–mineralized wall rocks are the main purpose in the economic geology and mineral exploration. Investigation of ore mineralogy and paragenetic sequence provides useful data on ore-forming processes in deposits, because the typical characteristics of various types of ore deposits are reflected by their mineral assemblages (Craig and Vaughan, 1994; White and Hedenquist, 1995). Common methods are usually based on mineralography, petrography and alteration minerals assemblages to delineate various mineralized zones in porphyry deposits (Beane, 1982; Schwartz, 1947; Sillitoe, 1997; Berger et al., 2008). Lowell (1968) firstly proposed a conceptual model of lateral and vertical variations in mineralogy within alteration zones. Some similar models were developed related to potassic alteration usually situated in the center and deep parts of porphyry ore deposits based on this model (Sillitoe and Gappe, 1984; Cox and Singer, 1986; Melfos et al., 2002). Fluid inclusion and stable isotope studies are other methods to outline different mineralization phases based on thermometric and isotope element parameters along with other geological particulars (e.g., Boyce et al., 2007; Faure et al., 2002; Wilson et al., 2007). The drillhole data with a logging information containing mineralographical information, host rock changes and alteration is helpful to delineate the mineralization zones. Different geological interpretations could be presented to detect zone boundaries, which may also lead to different results because the elemental grade distribution may not be taken into consideration.

Non-Euclidian fractal geometry (Mandelbrot, 1983) is an important branch of
non-linear mathematical sciences, which is applied in various research fields of geosciences since the 1980s. The relationships between geology, geochemistry and mineralogical settings with spatial information can be researched by the methods based on fractal geometry (Afzal et al., 2011; Carranza, 2008, 2009). Bolviken et al. (1992) and Cheng et al. (1994) have shown that geochemical patterns of various elements have fractal dimensions. The concentration-area (C-A) model was proposed by Cheng et al. (1994) to recognize geochemical anomalies from backgrounds and calculate elemental thresholds of different geochemical data. Furthermore, there are many other fractal models proposed and applied in geochemical exploration work including number-size (N-S) fractal model proposed by Mandelbrot (1983) and Agterberg (1995), power spectrum-area (S-A) fractal model proposed by Cheng et al. (1999), concentration-distance (C-D) fractal model proposed by Li et al. (2003), concentration-volume (C-V) fractal model proposed by Afzal et al. (2011) and power spectrum-volume (S-V) fractal model proposed by Afzal et al. (2012).

Methods of fractal analysis also serve to illustrate relationships of geological, geochemical and mineralogical settings with spatial information derived from analysis of mineral deposit occurrence data (Carranza, 2008; Carranza et al., 2009; Goncalves et al., 2001). Various geochemical processes can be described based on differences in fractal dimensions obtained from analysis of relevant geochemical data. Afzal et al. (2011) considered that the log-log plots obtained by fractal methods are useful tools to delineate different geological populations of geochemical data and the thresholds could be determined as some break points in those plots.

The application of fractal models to delineate various grade mineralization zones was dependent on the relationships between metal grades and volumes (Afzal et al., 2011; Agterberg et al., 1993; Cheng, 2007; Simet al., 1999; Turcotte, 1986). Afzal et al. (2011 and 2012) proposed concentration-volume (C-V) and power spectrum-volume (S-V) fractal model to delineate different porphyry-Cu mineralized zones and barren host rocks. In this paper, N-S, C-V and S-V fractal models were applied to delineate various mineralized zones and barren host rocks in the Pulang porphyry copper deposit, Yunnan, SW China.
2. Fractal models

2.1. Number-size (N-S) fractal model

Number-size (N-S) method proposed by Mandelbrot (1983) can be utilized to describe the distribution of geochemical populations (Sadeghi et al., 2012). In this method, geochemical data do not undergo any pre-processing (Mao et al., 2004). This model shows a relationship between desirable attributes (e.g. Cu concentration in this study) and their cumulative number of samples (Sadeghi et al., 2012). A power-law frequency model has been proposed to explain the N-S relationship according to the frequency distribution of elemental concentrations and cumulative number of samples with those attributes (e.g., Li et al., 1994; Sadeghi et al., 2012; Sanderson et al., 1994; Shi and Wang, 1998; Turcotte, 1996; Zuo et al., 2009a).

The N-S model proposed by Mandelbrot (1983) has been expressed as follows:

\[ N(\geq \rho) = F \rho^{-D} \]  

(1)

where \( \rho \) denotes element concentration, \( N(\geq \rho) \) denotes cumulative number of samples with concentration values greater than or equal to \( \rho \), \( F \) is a constant and \( D \) is the scaling exponent or fractal dimension of the distribution of element concentrations. According to Mandelbrot (1983), log-log plots of \( N(\geq \rho) \) versus \( \rho \) show straight line segments with different slopes -D corresponding to different concentration intervals.

2.2. Concentration-volume (C-V) fractal model

Afzal et al. (2011) proposed concentration-volume (C-V) fractal model based on the same idea as the concentration-area (C-A) model (Cheng et al., 1994) to analysis the relationship between the concentration of ore elements and accumulative volume with concentration greater than or equal to the given value (Afzal et al., 2011; Zuo et al., 2016; Lin et al., 2013; Sadeghi et al., 2012; Soltani et al., 2014; Sun and Liu, 2014; Wang, G. et al., 2012). It can be expressed as:

\[ V(\rho \leq \upsilon) \propto \rho^{-a_1}; V(\rho \geq \upsilon) \propto \rho^{-a_2} \]  

(2)

\( V(\rho \geq \upsilon) \) and \( V(\rho \leq \upsilon) \) represent those occupied volumes with concentrations above or equal to and less than or equal to the contour value \( \upsilon \); \( \upsilon \) indicates the threshold value of
a zone; $a_1$ and $a_2$ are the characteristic indexes. Thresholds obtained by this method indicate the boundaries between different grade mineralization zones and barren host rocks of ore deposits. The drillhole data of elemental concentration values were interpolated by using geostatistical estimation to compute $V(\rho \geq \upsilon)$ and $V(\rho \leq \upsilon)$, which are the volume values enclosed by a contour level $\rho$ in a 3D model.

2.3. Power spectrum-volume (S-V) fractal model

Different geochemical patterns in the spatial domain could be seen as layered signals of various frequencies. Cheng et al. (1999) proposed the power spectrum-area (S-A) fractal model to recognize geochemical anomalies from backgrounds utilizing the method of spectrum analysis in frequency domain according to this argument. This model is combined with concentration-area (C-A) model (Cheng et al. 1994). It offers an useful tool to determine an optimum threshold value between various patterns based on the scaling property.

Afzal et al. (2012) proposed the power spectrum-volume (S-V) fractal model to delineate different grade mineralization zones based on the same idea as the S-A model proposed by Cheng et al. (1999). S-V model was utilized in frequency domain. And it was performed by applying the fast Fourier transformation for assay data. The straight lines obtained by log-log plots indicate the relationships between power spectrums and relevant volumes of ore elements. They were utilized to recognize the hypogene zones and supergene enrichment zones from barren host rocks and leached zone of the deposit. The recognition of various mineralization zones is on the basis of the power-law relationships between power spectrums and occupied volumes. The formula is as follows:

$$V(\geq S) \propto S^{-2/\beta}$$  \hspace{1cm} (3)

Where, the power-law relationships between power spectrums ($S=||F(W_x, W_y, W_z)||$) and occupied volumes with power spectrums greater than or equal to $S$ can be indicated by this form; $F$ represents the fast Fourier transformation of the measurement $\mu(x, y, z)$; $W_x$, $W_y$ and $W_z$ respectively indicate wave numbers or angular frequencies in X, Y and Z axes directions on a 3D model. The range of index $\beta$ is $0<\beta\leq2$ or $1\leq2/\beta$.
with the special case of $\beta=2$ or $2/\beta=1$ corresponding to non-fractal or monofractal and $1<2/\beta$ to multifractals (Cheng, 2006).

By using the method of geostatistical estimation, the drillhole data of elemental concentration values were interpolated to construct the block model with ore element distribution. The power spectrum values can be obtained by using 3D fast Fourier transformation for ore element grades. The logarithm of all power spectrum values and accumulative volume values were calculated. And the log-log plot between power spectrums and volumes was drew according to previous counted values. Then the filters were constructed on the basis of threshold values obtained by the log-log plot of S-V. Finally, the power spectrums were converted back to the space domain by utilizing inverse fast Fourier transformation.

3. Geological setting of the Pulang porphyry copper deposit

The Pulang porphyry copper deposit is situated in the southern end of the Yidun continental arc, southwest China (Fig. 1). The continental arc was produced due to the westward subduction of Garze–Litang oceanic crust (Deng et al., 2014b, 2015; Wang et al., 2014). And the Pulang ore deposit, one of the largest porphyry copper deposits in China (Deng et al., 2012, 2014a; Mao et al., 2012, 2014), is characterized by typical porphyry-type alteration zone. The geological characteristics of the deposit, including the alteration types and their zonation, the geometry of orebody, metallogenic time and the geodynamic settings have been systematically researched (Leng et al., 2012; Li et al., 2011, 2013). The deposit consists of five ore-bearing porphyry bodies, covering an area of approximately 9 km$^2$, and the explored ore tonnage of Cu is estimated to be 6.50 Mt (Liu et al., 2013).

The outcrop strata of Pulang deposit are dominated by Upper Triassic Tumugou Formation clastic rocks and andesite, and Quaternary sediments (Fig. 1c). The Triassic porphyry intrusions primarily comprise quartz diorite porphyry, quartz monzonite porphyry, quartz diorite porphyrite and granodiorite porphyry. The Tumugou Formation strata was intruded by the quartz diorite porphyry with an age of 219.6 ± 3.5 Ma (Zircon U-Pb dating) (Pang et al., 2009). Then quartz monzonite porphyry with an age
of 212.8 ± 1.9 Ma and granodiorite porphyry with an age of 206.3 ± 0.7 Ma (Zircon U-Pb dating) (Liu et al., 2013) crosscut quartz diorite porphyry, respectively. The quartz monzonite porphyry is considered to be associated with mineralization because its age is similar with the molybdenite Re-Os isochron age of 213 ± 3.8 Ma from orebody (Zeng et al., 2004). Moreover, the Cu concentrations of quartz monzonite porphyry are higher than the other porphyries.

The porphyry-type alteration zones transit upward and outward from early potassium-silicate, through quartz-sericite to propylitization from the core of the quartz monzonite porphyry (Fig. 4). The wall rocks near the porphyries were mostly changed into hornfels. Systematic drilling has demonstrated that the potassium-silicate and quartz-sericite zones host the main orebodies, constituting the core of mineralized zones. And the propylitic zones and hornfels only develop the weak mineralization. The orebodies occur mainly in potassium-silicate and quartz-sericite, and occur as veins in the propylitic zones and hornfels. Major rock types in the deposit are quartz monzonite porphyry, quartz diorite porphyrite, granite diorite porphyry, quartz diorite porphyry and hornfels (Fig. 2). Metallic minerals mainly include pyrite, chalcopyrite with small amount of molybdenite and pyrrhotite (Fig. 3).

4. Fractal modeling

Based on the geological data (which include collar coordinates of each drillhole, azimuth and dip (orientation), lithology and mineralogy) recorded from 130 drillholes in Pulang deposit, 20492 lithogeochemical samples have been collected at 2 m intervals. The laboratory of the 3rd Geological Team of the Yunnan Bureau of Geology and Mineral Resources utilized the iodine-fluorine and oscillo-polarographic method to analyze the concentrations of Cu and associated paragenetic elements and its analytical uncertainty is less than 7% (Yunnan Diqing Nonferrous Metal Co. Ltd., 2009). Only Cu concentrations were researched in this study. The distribution of Cu concentrations is log-normal (Fig. 5). The experimental semi-variogram of Cu data of Pulang deposit indicates a range and nugget effect of 320.0m and 0.25, seperately (Fig. 6). The spherical model is fitted in regard to the experimental semi-variogram. The 3D model
of Cu concentrations distribution of Pulang deposit was produced with ordinary kriging method using the Geovia Surpac software on the basis of the semi-varioigram and anisotropic ellipsoid. Fundamentally, the accuracy of the interpolation results mainly depends on whether the interpolation model could well fit the spatial distribution characteristics of the deposit. Ordinary kriging was used because it is compatible with a stationary model; it only involves a variogram, and it is in fact the form of kriging used most (Chilès and Delfiner, 1999). Goovaerts (1997) showed that the values in un-sampled locations are estimated by the ordinary kriging method according to moving average of the interest variables satisfying various distribution forms of data. It is a spatial estimation method where the error variance is minimized. This error variance is based on the configuration of the data and its variogram (Yamamoto, 2005). The correct variogram in kriging interpolation can guarantee the accuracy of the interpolation results.

The accuracy of the spatial interpolation analysis is verified by comparing the difference between the measured values and the predicted values, so as to select the best variogram model. In order to test the variogram model, the cross-validation method was used to determine whether the parameters of the variogram model are correct. The distribution of the residual is normal (Fig.7) and the mean of error between the actual and estimated Cu grade values is equal to 0 (Table 1). It indicates that this model is reasonable, and the variogram parameters are unbiased for estimating the Cu grade.

The obtained block models were used as input to the fractal models. The Pulang deposit was modeled by 20m×20m×5m voxels and they were decided by the grid drilling dimensions and geometrical properties of the deposit (David, 1970). The Pulang deposit is totally modeled with 150,973 voxels. The terms of “highly”, “moderately” and “weakly” have been used to classify the mineralized zones based on fractal modeling and accordance with the classification of in terms of ore grades in the deposit.

4.1 Number-size (N-S) fractal modeling
The N-S model was applied to the Cu data (Fig. 8). The selection of breakpoints as threshold values appears to be an objective decision because geochemical populations are defined by different line segments in the N-S log-log plot. The straight fitted lines were obtained based on least-square regression (Agterberg et al., 1996; Spalla et al., 2010). In other words, the intensity of element enrichment is depicted by each slope of the line segment in the N-S log-log plots (Afzal et al., 2010; Bai et al., 2010).

Based on the classification of the 3D model of Cu data and the thresholds obtained from N-S fractal model (Table 2), highly mineralized zones are situated in the southern and central parts of Pulang deposit that coincide with the potassium-silicate alterations. However, small highly mineralized zones are located in the central parts of the Pulang deposit (Fig. 9). Moderately mineralized zones are disposed in a northwest-southeast trend correlated with phyllic zones. Weakly mineralized zones and barren host rocks are situated in the marginal parts of the area.

4.2. Concentration-volume (C-V) fractal modeling

The occupied volume values corresponding to Cu grades were computed to obtain the concentration-volume model according to the 3D model of Pulang deposit. Through the obtained C-V log-log plot, the threshold values of Cu grades were determined (Fig. 10). It indicates the power-law relationships between Cu grades and volumes. According to these results (Table 3), the low concentration zones exist in many parts of the deposit and are disposed along the NW-SE trend. Moderately and highly mineralized zones are situated in several parts of the center and south of the deposit (Fig. 11).

4.3. Power spectrum-volume (S-V) fractal modeling

Based on the geological data (which include collar coordinates of each drillhole, azimuth and dip (orientation), lithology and mineralogy) recorded from 130 drillholes in the deposit, a 3D model and block model of the distribution of Cu in Pulang deposit were constructed with ordinary kriging using the Geovia Surpac software.

The power spectrum (S) were calculated for the 3D elemental distribution using 3D fast Fourier transformation by MATLAB (R2016a). The logarithmic values of
power spectrums and relevant volume values were plotted against each other (Fig. 12). The straight lines fitted through log-log plot indicate different relationships between power spectrums and occupied volumes. The thresholds of logS=7.81 and logS=8.70 were decided by the log-log S-V plot. The 3D filters were designed to separate different mineralization zones on the basis of these threshold values. Inverse fast Fourier transformation was used to convert the decomposed components back into the space domain by MATLAB (R2016a). According to the results, Cu concentrations of the hypogene zones range from 0.23% to 1.33% (Table 4), and values of >1.33% Cu refer to the supergene enrichment zones, whereas values of <0.23% Cu pertain to the leached zone and barren host rocks (Fig. 13).

5. Comparison of fractal models and geological model of the deposit

Alteration models have a key role in zone delineation and also in presenting geological models, as described by Lowell and Guilbert (1970). The potassic and phyllic alterations control major mineralization within supergene enrichment and hypogene zones according to these models. Models of Cu mineralization zones derived via the fractal models can be compared with geological data in order to validate the results of analysis in different porphyry Cu deposits. Results of fractal modeling of Pulang deposit were compared with the 3D geological model of the deposit constructed by using the Geovia Surpac software and drillhole data (Fig. 2). Moreover, the results obtained from these fractal models are controlled by mineralogical investigations.

Carranza (2011) has illustrated an analysis for calculation of spatial correlations between two binary especially mathematical and geological models. An intersection operation between the mineralization zones obtained from fractal models and different alteration zones in the geological model was performed to derive the amount of voxels corresponding to each of the classes of overlap zones (Table 5). Using the obtained numbers of voxels, Type I error (T1E), Type II error (T2E), and overall accuracy (OA) of the fractal model were estimated with respect to different alteration zones due to geological data (Carranza, 2011). The values of OA of fractal models of mineralized zones were compared with each other as follows.
A comparison between highly mineralized zones based on the fractal models and potassic alteration zones resulted from the 3D geological model shows that there is a similarity among these fractal models. Overall accuracies for the C-V, N-S and S-V models are 0.50, 0.51 and 0.52, respectively (Table 6), which indicate that the S-V model gives better results to identify highly mineralized zones in the deposit. Because the fact that the number of overlapped voxels (A) in the S-V model is higher than those in N-S and C-V model. The correlation (from OA results) between highly mineralized zones obtained from S-V modeling and the potassic alteration zones is better than the N-S and C-V model because of a strong proportional relationship between extension and positions of voxels in the S-V model and potassic alteration zones in the 3D geological model.

Comparison between phyllic alteration zones resulted from the 3D geological model and moderately and weakly mineralized zones from fractal modeling shows that overall accuracies of the C-V, N-S and S-V fractal models with respect to phyllic alteration zones of the geological model are 0.59, 0.56 and 0.54, respectively. Overall accuracy value of moderately and weakly mineralized zones obtained from C-V modeling is higher than the mineralized zones obtained from N-S and S-V modeling (Table 7). On the other hand, moderately mineralized zones defined by C-V modeling have overlap with the phyllic zones in the 3D geological model. However, the results of the C-V model are more accurate than those of the N-S and S-V model with respect to the phyllic zones in the 3D geological model.

It could be considered that there are spatial correlations between different modeled Cu zones and geological features such as alterations and mineralogy. Several samples were collected from different drill holes in different grade mineralization zones of Pulang deposit to validate the results of fractal models. They were analyzed by microscopic identification and XRF (X-ray Fluorescence Spectrometer). The PL-B82 sample was collected from the drill hole situated in the high grade mineralization zones. There are high chalcopyrite content and some molybdenite (Fig.16a). PL-B62 sample was collected from the drill hole situated in the moderate grade mineralization zones. There are low chalcopyrite content and some pyrrhotite content in polished section
(Fig.16b). PL-B74 sample was collected from the drillhole located at the weakly mineralized zones with lower chalcopyrite content and some pyrrhotite (Fig.16c and Fig.16d). Results obtained from mineralogy, microscopic identification and drillhole scanning by XRF of these samples indicates that Cu concentrations are 1.80%, 1.32% and 0.41% in PL-B82, PL-B62 and PL-B74 samples, respectively (Table 8).

6. Conclusions

In the many cases, drillhole logging is dealing with the lack of proper diagnosis of geological phenomenon and it can undermine delineation of mineralized zones because it depends on the interpretation of individual loggers, which is subjective and no two loggers have the same interpretations. However, the conventional geological modeling based on drillhole data is fundamentally important for ore body spatial structure understanding and mathematical applications. Grades of the ore elements are not observed in conventional methods of geological ore modeling while the variations in ore grades in a mineral deposit is an obvious and salient feature. Given the problems as mentioned above, using a series of new established methods based on mathematical analyses such as fractal modeling seems to be inevitable.

In this paper, the number-size (N-S), concentration-volume (C-V) and power spectrum-volume (S-V) fractal models were used to delineate and recognize various Cu mineralized zones of Pulang porphyry copper deposit in the south end of the Yidun continental arc, SW China. All the fractal models reveal high grade Cu mineralized zones are situated in the central and southern parts of the deposit. The Cu threshold values of highly mineralized zones are 1.45% and 1.88% based on the N-S and C-V fractal models. And the Cu threshold of supergene enrichment zones is 1.33% based on the S-V fractal model. Models of moderately mineralized zones contain 0.28-1.45% Cu according to the N-S model, and 1.48-1.88% Cu according to the C-V model. The hypogene zones contain 0.23-1.33% Cu according to the S-V model. The N-S model reveals weakly mineralized zones and barren host rocks containing <0.28% Cu. In contrast, the C-V model reveals that barren host rocks contain <0.25% and weakly mineralized zones contain 0.25-1.48% Cu. And the S-V model reveals that barren host
rock and leached zone contain <0.23% Cu.

The comparison between highly mineralized zones based on the fractal models and potassic zones resulted from 3D geological model illustrates that the S-V fractal model is better than the N-S and C-V model because the fact that the number of overlapped voxels (A) in the S-V model is higher than those in the N-S and C-V model. Overall accuracies for the C-V, N-S and S-V models are 0.50, 0.51 and 0.52, respectively (Table 6), which indicate that the S-V model gives the best results to identify highly mineralized zones in the deposit. On the other hand, the correlation (from OA results) between highly mineralized zones obtained from S-V modeling and the potassic alteration zones is better than the N-S and C-V model because of a strong proportional relationship between extension and positions of voxels in the S-V model and potassic alteration zones in the 3D geological model.

Comparison between phyllic alteration zones resulted from the 3D geological model and moderate grade mineralization zones obtained from fractal models indicates that OA values of C-V, N-S and S-V fractal methods in regard to phyllic alteration zones of the geological model are 0.59, 0.56 and 0.54, respectively. Overall accuracy of moderately and weakly mineralized zones obtained from C-V modeling is higher than the mineralized zones obtained from N-S and S-V modeling (Table 7).

According to the correlation between the results driven by fractal modeling and geological logging from drillholes in the Pulang porphyry copper deposit, high grade mineralization zones generated by fractal models, especially the S-V model, has a better correlation with potassic alteration zones resulted from the 3D geological model than N-S and C-V model. The highly and moderately mineralized zones obtained from fractal models are both situated in the southern and central parts of the Pulang deposit that coincide with potassic and phyllic alteration zones. There is a better relationship between moderately and weakly mineralized zones derived by the C-V model and the phyllic alteration zones according to the 3D geological model than the N-S and S-V model.
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Fig.1. Geological map of the Pulang porphyry copper deposit, SW China. Modified after Yunnan Diqing Nonferrous Metal Co. Ltd., 2009.

Fig.2. Geological 3D models including lithology, alteration and 3D drill hole plot with the legend of each in the Pulang porphyry copper deposit. (Scale is in m$^3$.)

Fig.3. Photographs of alteration and mineralization in the Pulang porphyry copper deposit, SW China.

(a) Quartz monzonite porphyry with potassium-silicate alteration; (b) Quartz diorite porphyryite with quartz-sericite alteration; (c) Quartz diorite porphyrite with propylitic alteration; (d) Hornfels. Qtz=quartz; Pl=plagioclase; Kfs=K-feldspar; Bt=biotite; Ser=sericite; Chl=chlorite; Ep=epidote; Py=pyrite; Ccp=chalcopyrite; Mo=molybdenite; Po=pyrrhotite.

Fig.4. Cross section along exploration line 0 in the Pulang porphyry copper deposit, SW China. Modified after Wang et al., 2012.

Fig.5. Histograms of the Cu raw (a) and logarithmic transformation (b) data in the Pulang deposit.

Fig.6. The experimental semi–variogram (omni-directional) of Cu data in Pulang deposit.

Fig.7. The cross-validation results: (a) residual VS Cu grade; (b) the residual distribution histogram.

Fig.8. N–S log–log plot for Cu concentrations in the Pulang deposit.

Fig.9. Zones in the Pulang deposit based on thresholds defined from the N–S fractal model of Cu data: (a) highly mineralized zones; (b) moderately mineralized zones; (c) weakly mineralized zones and barren host rocks. (Scale is in m$^3$.)

Fig.10. C–V log–log plot for Cu concentrations in the Pulang deposit.

Fig.11. Zones in the Pulang deposit based on thresholds defined from the C–V fractal model of Cu data: (a) highly mineralized zones; (b) moderately mineralized zones; (c) weakly mineralized zones; (d) barren host rock. (Scale is in m$^3$.)

Fig.12. S–V log–log plot for Cu concentrations in the Pulang deposit.

Fig.13. Zones in the Pulang deposit based on thresholds defined from the S–V fractal model of Cu data: (a) the supergene enrichment zones; (b) the hypogene zones; (c) the leached zone and barren host rock. (Scale is in m$^3$.)

Fig.14. Highly mineralized zones in the Pulang deposit: (a) potassium-silicate zone resulted from the 3D geological model from drillhole geological data; (b) N–S modeling of Cu data; and (c) C–V modeling of Cu data; (d) S–V modeling of Cu data. (Scale is in m$^3$.)

Fig.15. Moderately mineralized zones in the Pulang deposit: (a) quartz–sericite zones resulted from the 3D geological model from drillhole geological data; (b) N–S modeling of Cu data; and (c) C–V modeling of Cu data; (d) S–V modeling of Cu data. (Scale is in m$^3$.)

Fig.16. Chalcopyrite content in several samples based on mineralogical study: (a) PL-B82 sample was collected from the drillhole situated in the high grade mineralization zones.; (b) PL-B62 sample was collected from the drillhole situated in the moderately grade mineralization zones.; (c) and (d) PL-B74 sample was collected from the drillhole located at the weakly mineralized zones.
Table 1 The results of statistical characteristics of the residual.
Table 2 Thresholds concentrations obtained by using N-S model based on Cu% in Pulang deposit.
Table 3 Thresholds concentrations obtained by using C-V model based on Cu% in Pulang deposit.
Table 4 Ranges of power spectrum (S) for different mineralization zones in Pulang deposit.
Table 5 Matrix for comparing performance of fractal modeling results with geological model. A, B, C, and D represent number of voxels in overlaps between classes in the binary geological model and the binary results of fractal models (Carranza, 2011).
Table 6 Overall accuracy (OA), Type I and Type II errors (T1E and T2E, respectively) with respect to potassic alteration zone resulted from geological model and threshold values of Cu obtained through C–V, N–S and S–V fractal modeling.
Table 7 Overall accuracy (OA), Type I and Type II errors (T1E and T2E, respectively) with respect to phyllic alteration zone resulted from geological model and threshold values of Cu obtained through C–V, N–S and S–V fractal modeling.
Table 8 Results of XRF analysis of samples collected from different mineralized zones in the Pulang porphyry copper deposit.
Fig. 1.
Fig. 2.
Fig. 3.

Fig. 4.
Fig. 8.
Fig. 9

Fig. 10.
Fig. 11.
Fig. 12.

(a)

Fig. 13.

(b)

(c)
Fig. 14.
Fig. 15.
Fig. 16.
### Table 1

<table>
<thead>
<tr>
<th>Variables</th>
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<td>Mean</td>
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<td>Variance</td>
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<td>Standard Deviation</td>
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### Table 2

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<th>Mineralized zones</th>
<th>Thresholds (Cu%)</th>
<th>Range (Cu%)</th>
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<tr>
<td>Barren host rock</td>
<td>&lt;0.28</td>
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<tr>
<td>Weakly mineralized</td>
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<td>0.28-1.45</td>
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<tr>
<td>Moderately mineralized</td>
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<td>1.45</td>
<td>&gt;1.45</td>
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### Table 3

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<th>Mineralized zones</th>
<th>Thresholds (Cu%)</th>
<th>Range (Cu%)</th>
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<tr>
<td>Barren host rock</td>
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<td>1.48</td>
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### Table 4

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<th>Mineralized zones</th>
<th>PS threshold</th>
<th>Range of PS</th>
<th>Range (Cu%)</th>
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<td>leached zone and barren host rock</td>
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<td>hypogene zones</td>
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<tr>
<td>supergene enrichment zones</td>
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### Table 5

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<th>Geological model</th>
<th>Inside zone</th>
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<td>True positive (A)</td>
<td>False positive (B)</td>
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<tr>
<td>Inside zone</td>
<td>False negative (C)</td>
<td>True negative (D)</td>
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<tr>
<td>Outside zone</td>
<td>Type I error = C/(A+C)</td>
<td>Type II error = B/(B+D)</td>
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<td>Overall accuracy = (A+D)/(A+B+C+D)</td>
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### Table 6

<table>
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<tr>
<th>Fractal Model</th>
<th>Inside zones</th>
<th>Outside zones</th>
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<td>C–V fractal model of highly mineralized zones</td>
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<td>Outside zones</td>
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<td>B 1360</td>
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### Table 7

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### Table 8

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<tr>
<th>Sample no.</th>
<th>Mineralized zones obtained by fractal models</th>
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<td>PL-B74</td>
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<td>PL-B82</td>
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<td>1.80</td>
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